Name _____ Date_____

Principles of Counting

Objective: To find the total possible number of arrangements (ways) an event may occur.

a) Identify **the number of parts** (Area Codes, Zip Codes, License Plates, Password, Short Melodies)
b) Start with **the most restricted part** and write the number of possible choices.
c) Write the **number of choices** for other parts
d) Multiply all the numbers.

1) How many different **PIN** numbers codes are possible? D D D D = $10\times10\times10\times10=10,000$

2) How many different **ZIP** codes are possible? D D D D D = $10\times10\times10\times10\times10=100,000$

3) How many different **ZIP** codes are possible with no zero at the beginning?

D D D D D = $9\times10\times10\times10\times10=90,000$

4) How many different 7- part license plates are possible with one digit first, 3 letters after followed by another 3 digits?

D L L L D D D = $10\times26\times26\times26\times10\times10\times10=175,760,000$

5) How many different 7- part license plates are possible if each part can use letter or digit?

D L L L D D D = $36\times36\times36\times36\times36\times36\times36=78,364,164,096$

6) How many different 6-part password can be written (case sensitive with 10 digits, 52 letters and 8 symbols)

$70\times70\times70\times70\times70\times70=117,649,000,000$

7) How many different types of pizza with two toppings can we order, if we have 4 choices of size, two choices of thickness, and 8 choices of toppings. **S Th T1 T2** $4\times2\times8\times7=448$

8) How many different types of pizza with three toppings can we order, if we have 4 choices of size, two choices of thickness, and 8 choices of toppings. **S Th T1 T2 T3**

$4\times2\times8\times7\times6=2,688$

9) How many different 8-note melodies can be made by a 44-key keyboard?

$44^8=14,048,223,625,220$

10) How many different 4- digit even numbers can we write with (0,5,6,3,8,7)?

D D D D

$5\times6\times6\times3=540$

Hint: To be 4- digit **zero can not be used** as the first digit, and to be an even number the **last number** can be 0,6,8, that give us 3 choices.

Name _____ Date_____

Worksheet A2 : Fundamental Counting Principle, Factorials, Permutations Intro

1. A restaurant offers four sizes of pizza, two types of crust, and eight toppings. How many possible combinations of pizza with one topping are there?

2. How many ways can 5 paintings be line up on a wall?

3. Rob has 4 shirts, 3 pairs of pants, and 2 pairs of shoes that all coordinate. How many outfits can you put together?

4. Grace loves to eat salad! How many salads can she put together if she can pick out one type of lettuce from 2 choices, one vegetable from 4 choices and one dressing from 7 choices?

5. PA license plates have 3 letters followed by 4 numbers.
   a. If the same letter or number can be repeated, how many can be made?

   b. If the same letter CANNOT be repeated, how many can be made?

6. How many 5-digit numbers can be formed (using 0 – 9)?

7. How many 5-digit numbers can be formed if each one uses all the digits 0, 1, 2, 3, 4 without repetition?

8. In how many ways can 6 bicycles be parked in a row?

Counting Examples

Factorial Numbers $n!$

$1! = 1$ $\quad 2! = 2 \times 1 = 2$ $\quad 3! = 3 \times 2 \times 1 = 6$ $\quad 4! = 4 \times 3 \times 2 \times 1 = 24$ $\quad 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

Permutation:

$nPx = \dfrac{n!}{(n-x)!}$ **Selecting x out of n when <u>order in selection matters</u> and repetition is not allowed**

$4P2 = \dfrac{4!}{(4-2)!} = \dfrac{4!}{2!} = \dfrac{24}{2} = 12$ $\qquad 5P3 = \dfrac{5!}{(5-3)!} = \dfrac{5!}{2!} = \dfrac{120}{2} = 60$

Find $6P2 =$ ———— $=$ $\qquad 5P2 =$ ———— $=$ $\qquad 6P4 =$ ———— $=$

Combination:

$nCx = \dfrac{n!}{x!\,(n-x)!}$ **Selecting x out of n when <u>order in selection does not matter</u> and repetition is not allowed**

$4C2 = \dfrac{4!}{2!(4-2)!} = \dfrac{4!}{2!2!} = \dfrac{24}{4} = 6$ $\qquad 5C3 = \dfrac{5!}{3!(5-3)!} = \dfrac{5!}{3!2!} = \dfrac{120}{6 \times 2} = 10$

Find $6C2 =$ ———— $=$ $\qquad 5C2 =$ ———— $=$ $\qquad 6C4 =$ ———— $=$

Determine whether each of the following situations is a Combination or Permutation.

1. Creating an access code for a computer site using any 8 alphabet letters.

2. Determining how many different ways you can elect a Chairman and Co-Chairman of a committee if you have 10 people to choose from.

3. Voting to allow 10 new members to join a club when there are 25 that would like to join.

4. Finding different ways to arrange a line-up for batters on a baseball team.

5. Choosing 3 toppings for a pizza if there are 9 choices.

MORE COMBINATION AND PERMUTATION PRACTICE PROBLEMS:

1. Suppose that 7 people enter a swim meet. Assuming that there are no ties, in how many ways could the gold, silver, and bronze medals be awarded?

2. How many different committees of 3 people can be chosen to work on a special project from a group of 9 people?

3. A coach must choose how to line up his five starters from a team of 12 players. How many different ways can the coach choose the starters?

4. John bought a machine to make fresh juice. He has five different fruits: strawberries, oranges, apples, pineapples, and lemons. If he only uses two fruits, how many different juice drinks can John make?

5. How many different four-letter passwords can be created for a software access if no letter can be used more than once?

6.How many different ways you can elect a Chairman and Co-Chairman of a committee if you have 10 people to choose from.

7. There are 25 people who work in an office together. Five of these people are selected to go together to the same conference in Orlando, Florida. How many ways can they choose this team of five people to go to the conference?

8. There are 25 people who work in an office together. Five of these people are selected to attend five different conferences. The first person selected will go to a conference in Hawaii, the second will go to New York, the third will go to San Diego, the fourth will go to Atlanta, and the fifth will go to Nashville. How many such selections are possible?

9. John couldn't recall the Serial number on his expensive bicycle. He remembered that there were 6 different digits, none used more than once, but couldn't remember what digits were used. He decided to write down all of the possible 6 digit numbers . How many different possibilities will he have to create?

10. How many different 7-card hands can be chosen from a standard 52-card deck?

11. One hundred twelve people bought raffle tickets to enter a random drawing for three prizes. How many ways can three names be drawn for first prize, second prize, and third prize?

12. A disc jockey has to choose three songs for the last few minutes of his evening show. If there are nine songs that he feels are appropriate for that time slot, then how many ways can he choose and arrange to play three of those nine songs?

Answers:

1. ${}_7P_3 = 210$

2. ${}_9C_3 = 84$

3. ${}_{12}P_5 = 95{,}040$

4. ${}_5C_2 = 10$

5. ${}_{26}P_4 = 358{,}800$

6. ${}_{10}P_2 = 90$

7. ${}_{25}C_5 = 53{,}130$

8. ${}_{25}P_5 = 6{,}375{,}600$

9. ${}_{10}P_6 = 151{,}200$

10. ${}_{52}C_7 = 133{,}784{,}560$

11. ${}_{112}P_3 = 1{,}367{,}520$

12. ${}_9P_3 = 504$