Applications of Systems of Linear Equations

Objectives
1. Solve geometry problems by using two variables.
2. Solve money problems by using two variables.
3. Solve mixture problems by using two variables.
4. Solve distance–rate–time problems by using two variables.
5. Solve problems with three variables by using a system of three equations.

PROBLEM-SOLVING HINT
When solving an applied problem using two variables, it is a good idea to pick letters that correspond to the descriptions of the unknown quantities. For example above, we could choose \( c \) to represent the number of citrons and \( w \) to represent the number of wood apples.

Applications of Systems of Linear Equations

Solving an Applied Problem by Writing a System of Equations

**Step 1** Read the problem, several times if necessary. What information is given? What is to be found? This is often stated in the last sentence.

**Step 2** Assign variables to represent the unknown values. Use a sketch, diagram, or table, as needed.

**Step 3** Write a system of equations using the variable expressions.

**Step 4** Solve the system of equations.

**Step 5** State the answer to the problem. Label it appropriately. Does it seem reasonable?

**Step 6** Check the answer in the words of the original problem.

Classroom Example 1 Finding the Dimensions of a Soccer Field

A rectangular soccer field has perimeter 360 yd. Its length is 20 yd more than its width. What are its dimensions?

**Solution:**

**Step 1 Read** the problem again. We are asked to find the dimensions of the field.

**Step 2 Assign variables.** Let \( L = \) the length and \( W = \) the width.

**Step 3 Write a system of equations** using the variable expressions.

**Step 4 Solve** the system of equations.

**Step 5 State the answer** to the problem. Label it appropriately. Does it seem reasonable?

**Step 6 Check** the answer in the words of the original problem.

The system is

\[
\begin{align*}
L &= W + 20 \quad (1) \\
2W + 2L &= 360 \quad (2)
\end{align*}
\]

**Step 4 Solve.** Substitute \( W + 20 \) for \( L \) in equations (2).

\[
\begin{align*}
2W + 2(W + 20) &= 360 \\
2W + 2W + 40 &= 360 \\
4W &= 320 \\
W &= 80
\end{align*}
\]

Substitute \( W = 80 \) into equation (1).

\[
L = 80 + 20 = 100
\]
For the 2009 Major League Baseball and National Football League seasons, based on average ticket prices, three baseball tickets and two football tickets would have cost $229.90. Two baseball tickets and one football ticket would have cost $128.27. What were the average ticket prices for the tickets for the two sports? (Source: Team Marketing Report.)

**Step 1 Read** the problem again. There are two unknowns.

**Step 2 Assign variables.**

Let \( x \) = the average cost of baseball tickets, and \( y \) = the average cost of football tickets.

**Step 3 Write a system of equations.**

\[
\begin{align*}
3x + 2y &= 229.90 \quad (1) \\
2x + y &= 128.27 \quad (2)
\end{align*}
\]

**Step 4 Solve.**

Multiply equation (2) by \(-2\) and add to equation (1).

\[
\begin{align*}
3x + 2y &= 229.90 \quad (1) \\
-4x - 2y &= -256.54 \quad -2 \times (2)
\end{align*}
\]

Add the equations:

\[
-x = -26.64
\]

Let \( x = 26.64 \) in equation (2).

\[
53.28 + y = 128.27
\]

\[
y = 74.99
\]

**Step 5 State the answer.**

The average cost of a baseball ticket is $26.64 and the average cost of a football ticket is $74.99. The answer is correct.

**Step 6 Check.**

\[
3(26.64) + 2(74.99) = 229.90 \quad \text{and} \quad 2(26.64) + 74.99 = 128.27.
\]
A train travels 600 mi in the same time that a truck travels 520 mi. Find the speed of each vehicle if the train’s average speed is 8 mph faster than the truck’s.

Solution:

Step 1 Read the problem.
We need to find the speed of each vehicle.

Step 2 Assign variables.
Let $x =$ the train’s speed and $y =$ the truck’s speed.

Step 3 Write a system of equations.

<table>
<thead>
<tr>
<th></th>
<th>Distance</th>
<th>Rate</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Train</td>
<td>600</td>
<td>$x$</td>
<td>$\frac{600}{x}$</td>
</tr>
<tr>
<td>Truck</td>
<td>520</td>
<td>$y$</td>
<td>$\frac{520}{y}$</td>
</tr>
</tbody>
</table>

The times must be equal.

Step 4 Solve.
Substitute $y + 8$ for $x$ in equation (1) to find $y$.

\[
-520(x - 8) + 600y = 0 \quad (1)
\]

\[
-520y + 4160 + 600y = 0 
\]

\[
y = 52
\]

Since $y = 52$ and $x = y + 8$, $x = 60$.

Step 5 State the answer.
The train’s speed is 60 mph, the truck’s speed is 52 mph.

Step 6 Check.

\[
60 = 52 + 8
\]

It would take the train 10 hours to travel 600 miles at 60 mph, which is the same amount of time it would take the truck to travel 520 miles at 52 mph.

The answer is correct.

Solve problems with three variables by using a system of three equations.

PROBLEM-SOLVING HINT
If an application requires finding three unknown quantities, we can use a system of three equations to solve it. We extend the method used for two unknowns.

A department store display features three kinds of perfume: Felice, Vivid, and Joy. There are 10 more bottles of Felice than Vivid, and 3 fewer bottles of Joy than Vivid. Each bottle of Felice costs $8, Vivid costs $15, and Joy costs $32. The total value of the all the perfume is $589. How many bottles of each are there?

Solution:

Step 1 Read the problem.
There are 3 unknowns.

Step 2 Assign variables.
Let $x =$ the number of bottles of Felice at $8$

$y =$ the number of bottles of Vivid at $15$, and

$z =$ the number of bottles of Joy at $32$. 

Objective 5
Solve problems with three variables by using a system of three equations.
Step 3 Write a system of equations.
There are 10 more bottles of Felice, so \( x = y + 10 \). (1)
There are 3 fewer bottles of Joy than Vivid, so \( z = y - 3 \). (2)
The total value is $589, so \( 8x + 15y + 32z = 589 \). (3)

Step 4 Solve.
Substitute \( y + 10 \) for \( x \) and \( y - 3 \) for \( z \) in equation (3) to find \( y \).
\[
\begin{align*}
8(y + 10) + 15y + 32(y - 3) &= 589 \\
8y + 80 + 15y + 32y - 96 &= 589 \\
55y &= 605 \\
y &= 11
\end{align*}
\] Since \( y = 11 \), \( x = y + 10 = 21 \) and \( z = y - 3 = 8 \).

Step 5 State the answer.
There are 21 bottles Felice, 11 of Vivid, and 8 of Joy.
The answer is correct.

Step 6 Check.
\[
21(8) + 11(15) + 8(32) = 589
\]
The answer is correct.

Step 2 Assign variables.
Let \( x \) = the number of tons of newsprint
\( y \) = the number of tons of bond, and
\( z \) = the number of tons of copy machine paper.

Step 3 Write a system of equations.
\[
\begin{align*}
3x + 2y + 2z &= 4200 \quad (1) \\
x + 4y + 3z &= 5800 \quad (2) \\
3y + 2z &= 3900 \quad (3)
\end{align*}
\]

Step 4 Solve the system to find \( x = 400 \), \( y = 900 \), and \( z = 600 \).
\[
\begin{align*}
3x + 2y + 2z &= 4200 \\
x + 4y + 3z &= 5800 \\
3y + 2z &= 3900
\end{align*}
\]

Step 5 State the answer.
The paper mill can make 400 tons of newsprint, 900 tons of bond, and 600 tons of copy machine paper.

Step 6 Check that these values satisfy the conditions of the problem.
The answer is correct.