5.3 Polynomial Functions, Graphs and Composition

Objectives

1 Recognize and evaluate polynomial functions.

2 Use a polynomial function to model data.

3 Add and subtract polynomial functions.

4 Find the composition of functions.

5 Graph basic polynomial functions.

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Recognize and evaluate polynomial functions.

Polynomial Function

A **polynomial function of degree *n*** is defined by

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_0,$$

for real numbers a_n, a_{n-1}, $\dots a_1$, and a_0, where $a_n \neq 0$ and n is a whole number.

CLASSROOM EXAMPLE 1 **Evaluating Polynomial Functions**

Let $f(x) = -x^2 + 5x - 11$. Find $f(-4)$

Solution:

Read this as "*f* of negative 4," not *f* times negative 4."

$f(-4) = -x^2 + 5x - 11$

$= -(-4)^2 + 5(-4) - 11$ **Substitute –4 for *x*.**

$= -16 - 20 - 11$ **Order of operations**

$= -47$ **Subtract.**

CLASSROOM EXAMPLE 2 **Using a Polynomial Model to Approximate Data**

The number of students enrolled in public schools (grades pre-K-12) in the United States during the years 1990 through 2006 can be modeled by the polynomial function defined by

$$P(x) = -0.01774x^2 + 0.7871x + 41.26,$$

where $x = 0$ corresponds to the year 1990, $x = 1$ corresponds to 1991, and so on, and $P(x)$ is in millions. Use this function to approximate the number of public school students in 2000.

(*Source:* Department of Education.)

Solution:

$P(x) = -0.01774x^2 + 0.7871x + 41.26$

$P(10) = -0.01774(10)^2 + 0.7871(10) + 41.26$

$P(10) = -1.774 + 7.87 + 41.26$

= 47.4 million students

Add and subtract polynomial functions.

The operations of addition, subtraction, multiplication, and division are also defined for functions.

For example, businesses use the equation "profit equals revenue minus cost," written in function notation as

$$P(x) = R(x) - C(x)$$

Profit function / **Revenue function** / **Cost function**

where *x* is the number of items produced and sold.

Add and subtract polynomial functions.

Adding and Subtracting Functions

If $f(x)$ and $g(x)$ define functions, then

$$(f + g)(x) = f(x) + g(x)$$ **Sum function**

and

$$(f - g)(x) = f(x) - g(x).$$ **Difference function**

In each case, the domain of the new function is the intersection of the domains of $f(x)$ and $g(x)$.

CLASSROOM EXAMPLE 3 — **Adding and Subtracting Functions**

For $f(x) = 3x^2 + 8x - 6$ and $g(x) = -4x^2 + 4x - 8$, find each of the following.

Solution:

$(f + g)(x)$

$= f(x) + g(x)$

$= (3x^2 + 8x - 6) + (-4x^2 + 4x - 8)$

$= -x^2 + 12x - 14$

$(f - g)(x)$

$= f(x) - g(x)$

$= (3x^2 + 8x - 6) - (-4x^2 + 4x - 8)$

$= 3x^2 + 8x - 6 + 4x^2 - 4x + 8$

$= 7x^2 + 4x + 2$

CLASSROOM EXAMPLE 4 — **Adding and Subtracting Functions**

For $f(x) = 18x^2 - 24x$ and $g(x) = 3x$, find each of the following.

$(f + g)(x)$ and $(f + g)(-1)$ $\qquad$ $(f - g)(x)$ and $(f - g)(1)$

Solution:

$(f + g)(x)$

$= f(x) + g(x)$

$= 18x^2 - 24x + 3x$

$= 18x^2 - 21x$

$(f - g)(x)$

$= f(x) - g(x)$

$= 18x^2 - 24x - 3x$

$= 18x^2 - 27x$

$(f + g)(-1)$

$= f(-1) + g(-1)$

$= [18(-1)^2 - 24(-1)] + 3(-1)$

$= [18 + 24] - 3$

$= 39$

$(f - g)(1)$

$= f(1) - g(1)$

$= [18(1)^2 - 24(1)] - 3(1)$

$= [18 - 24] - 3$

$= -9$

Find the composition of functions.

Composition of Functions

If f and g are functions, then the **composite function,** or **composition,** of g and f is defined by

$$(g \circ f)(x) = g(f(x))$$

for all x in the domain of f such that $f(x)$ is in the domain of g.

CLASSROOM EXAMPLE 5 — **Evaluating a Composite Function**

Let $f(x) = x - 4$ and $g(x) = x^2$. Find $(f \circ g)(3)$.

Solution:

Evaluate the "inside" function value first.

$(f \circ g)(3) = f(g(3))$

$= f(3^2)$

$= f(9)$

$= 9 - 4$

$= 5$

Now evaluate the "outside" function.

CLASSROOM EXAMPLE 6 — **Finding Composite Functions**

Let $f(x) = 3x + 6$ and $g(x) = x^3$. Find the following.

Solution:

$(f \circ g)(2)$ $\qquad$ $(g \circ f)(x)$

$(f \circ g)(2) = f(2^3)$

$= f(8)$

$= 3(8) + 6$

$= 30$

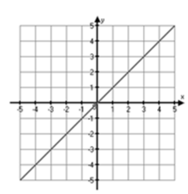
$(g \circ f)(x) = g(3x + 6)$

$= (3x + 6)^3$

Graph basic polynomial functions.

The simplest polynomial function is the **identity function,** defined by $f(x) = x$ and graphed below. This function pairs each real number with itself.

x	f(x) = x
–2	–2
–1	–1
0	0
1	1
2	2

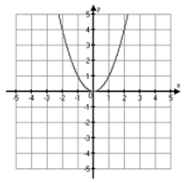


The domain (set of x-values) is all real numbers, $(-\infty, \infty)$.
The range (set of y-values) is also $(-\infty, \infty)$.

Graph basic polynomial functions.

Another polynomial function, defined by $f(x) = x^2$ and graphed below, is the **squaring function.** For this function, every real number is paired with its square. The graph of the squaring function is a ***parabola.***

x	*f*(*x*) = *x*²
–2	4
–1	1
0	0
1	1
2	4



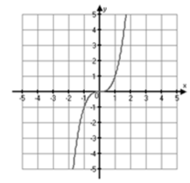
The domain is all real numbers, $(-\infty, \infty)$.

The range is $[0, \infty)$.

Graph basic polynomial functions.

The **cubing function** is defined by $f(x) = x^3$ and graphed below. This function pairs every real number with its cube.

x	*f*(*x*) = *x*³
–2	–8
–1	–1
0	0
1	1
2	8



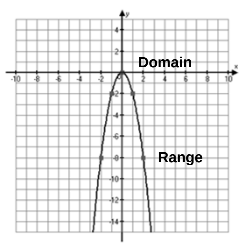
The domain and range are both $(-\infty, \infty)$.

CLASSROOM EXAMPLE 7 **Graphing Variations of Polynomial Functions**

Graph $f(x) = -2x^2$. Give the domain and range.

Solution:

x	*f*(*x*) = –2*x*²
–2	–8
–1	–2
0	0
1	–2
2	–8



The domain is $(-\infty, \infty)$.

The range is $(-\infty, 0]$.