Objectives

1. Use the product rule for exponents.
2. Define 0 and negative exponents.
3. Use the quotient rule for exponents.
4. Use the power rules for exponents.
5. Simplify exponential expressions.
6. Use the rules for exponents with scientific notation.

We use exponents to write products of repeated factors. For example, $2^5$ is defined as $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$. The number 5, the exponent, shows that the base 2 appears as a factor five times. The quantity $2^5$ is called an exponential or a power. We read $2^5$ as "2 to the fifth power" or "2 to the fifth."

### Product Rule for Exponents

If $m$ and $n$ are natural numbers and $a$ is any real number, then $a^m \cdot a^n = a^{m+n}$.

That is, when multiplying powers of like bases, keep the same base and add the exponents.

**CLASSROOM EXAMPLE 1 Using the Product Rule for Exponents**

Solution:

Be careful not to multiply the bases. Keep the same base and add the exponents.

Define 0 and negative exponents.

**Zero Exponent**

If $a$ is any nonzero real number, then $a^0 = 1$.

The expression $0^0$ is undefined.

### Zero Exponent

If $a$ is any nonzero real number, then $a^0 = 1$.

The expression $0^0$ is undefined.

**CLASSROOM EXAMPLE 2 Using 0 as an Exponent**

Evaluate.

Solution:

The number 5, the exponent, shows that the base 2 appears as a factor five times. The quantity $2^5$ is called an exponential or a power. We read $2^5$ as "2 to the fifth power" or "2 to the fifth."

$2^5$ is defined as $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$. The number 5, the exponent, shows that the base 2 appears as a factor five times. The quantity $2^5$ is called an exponential or a power. We read $2^5$ as "2 to the fifth power" or "2 to the fifth."

### Product Rule for Exponents

If $m$ and $n$ are natural numbers and $a$ is any real number, then $a^m \cdot a^n = a^{m+n}$.

That is, when multiplying powers of like bases, keep the same base and add the exponents.

**CLASSROOM EXAMPLE 1 Using the Product Rule for Exponents**

Apply the product rule, if possible, in each case.

Solution:

- $m^8 \cdot m^6 = m^{8+6} = m^{14}$
- Cannot be simplified further because the bases $m$ and $p$ are not the same. The product rule does not apply.
- $(-5p^4)(-9p^5) = (-5)(-9)(p^4p^5) = 45p^{4+5} = 45p^9$
- $(-3x^2y^3)(7x^4y^4) = (-3)(7)x^2x^4y^3y^4 = -21x^{2+4}y^{3+4} = -21x^6y^7$
Define 0 and negative exponents.

### Negative Exponent

For any natural number \( n \) and any nonzero real number \( a \),

\[
a^{-n} = \frac{1}{a^n}.
\]

A negative exponent does not indicate a negative number; negative exponents lead to reciprocals.

### Example 3

Using Negative Exponents

Write with only positive exponents.

Solution:

\[
6^{-5} = \frac{1}{6^5}
\]

\[
(2x)^{-4}, x \neq 0 = \frac{1}{(2x)^4}, x \neq 0
\]

\[
-7p^{-4}, p \neq 0 = -7 \left( \frac{1}{p^4} \right) = \frac{-7}{p^4}, p \neq 0
\]

Evaluate \( 4^{-1} - 2^{-1} \):

\[
\frac{1}{4} - \frac{1}{2} = \frac{1}{4} - \frac{2}{4} = \frac{-1}{4}
\]

### Example 4

Using Negative Exponents

Evaluate.

Solution:

\[
\frac{1}{4^{-3}} = \frac{1}{\left(\frac{1}{4}\right)^3} = 1 \div \frac{1}{4^3} = 1 \cdot 4^3 = 4^3 = 64
\]

\[
\frac{3^{-3}}{9^{-1}} = \frac{\frac{1}{3^3}}{\frac{1}{9}} = \frac{\frac{1}{27}}{\frac{1}{9}} = \frac{1}{27} \div \frac{1}{9} = \frac{3}{3} = 1
\]

### Example 5

Using the Quotient Rule for Exponents

Apply the quotient rule, if possible, and write each result with only positive exponents.

Solution:

\[
\frac{m^8}{m^{13}} = m^{8-13} = m^{-5}, m \neq 0
\]

\[
\frac{5^{-6}}{5^{12}} = 5^{-6-12} = 5^{-18}, 5 \neq 0
\]

\[
\frac{x^3}{y^7}, y \neq 0
\]

Cannot be simplified because the bases \( x \) and \( y \) are different. The quotient rule does not apply.
Use the power rules for exponents.

Power Rule for Exponents
If \(a\) and \(b\) are real numbers and \(m\) and \(n\) are integers, then

\[
\text{a) } (a^m)^n = a^{mn}, \quad \text{b) } (ab)^n = a^n b^n,
\]

and \(\text{c) } \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} (b \neq 0).\)

That is,

a) To raise a power to a power, multiply exponents.
b) To raise a product to a power, raise each factor to that power.
c) To raise a quotient to a power, raise the numerator and the denominator to that power.

CLASSROOM EXAMPLE 6
Using the Power Rules for Exponents
Simplify, using the power rules.

Solution:

\[
(r^3)^4 = r^{3 \cdot 4} = r^{12},
\]

\[
(-3y^2)^2 = (-3)^2 (y^2)^2 = 9y^{5+2} = 9y^7.
\]

\[
\frac{3^n}{4} = \frac{3^n}{4^n} = \frac{27}{64}.
\]

Use the power rules for exponents.

Special Rules for Negative Exponents, Continued
If \(a \neq 0\) and \(b \neq 0\) and \(n\) is an integer, then

\[
a^{-n} = \left(\frac{1}{a}\right)^n \quad \text{and} \quad \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n.
\]

That is, any nonzero number raised to the negative \(n\)th power is equal to the reciprocal of that number raised to the \(n\)th power.

CLASSROOM EXAMPLE 7
Using Negative Exponents with Fractions
Write with only positive exponents and then evaluate.

Solution:

\[
\left(\frac{2}{3}\right)^{-4} = \left(\frac{3}{2}\right)^4 = \frac{81}{16}.
\]

\[
\frac{1}{2x}^{-5}, \quad x \neq 0 = \left(\frac{2x}{1}\right)^5 = 2x^{-4} = 32x^4.
\]

Use the power rules for exponents.

Definition and Rules for Exponents
For all integers \(m\) and \(n\) and all real numbers \(a\) and \(b\), the following rules apply.

Product Rule \(a^m \cdot a^n = a^{m+n}\)
Quotient Rule \(\frac{a^m}{a^n} = a^{m-n} (a \neq 0)\)
Zero Exponent \(a^0 = 1 (a \neq 0)\)

Definition and Rules for Exponents, Continued

Negative Exponent \(a^{-x} = \frac{1}{a^x} (a \neq 0)\)

\[
(a^m)^n = a^{mn} \quad (ab)^n = a^n b^n
\]

Power Rules \(\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} (b \neq 0)\)

\[
\frac{1}{a^x} = a^{-x} (a \neq 0) \quad a^{-x} = b^{-x} (b, a \neq 0)
\]

Special Rules \(\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x} (a, b \neq 0)\)
**Objective 5**

Simplify exponential expressions.

---

**CLASSROOM EXAMPLE 8**

Using the Definitions and Rules for Exponents

Simplify. Assume that all variables represent nonzero real numbers.

Solution:

\[
4^{-5} = 4^{(5-5)} = 4^{10} = \frac{1}{4^{10}}
\]

\[
X^{-2} \cdot X^{-4} \cdot X^8 = X^{-2+(-4)+8} = X^2 = \frac{1}{X^4}
\]

\[
\left(\frac{m^2}{n}\right)^{-2} = \frac{(m^2)^{-2}}{n^{-2}} = \frac{m^{-4}}{n^{-2}} = \frac{m^4}{n^2}
\]

\[
= \frac{m^{4+(-3)}}{n^{-2-4}} = \frac{m^{1+3}}{n^{-2}} = \frac{m^4}{n^3}
\]

\[
= \frac{1}{m^3}
\]

---

**Use the rules for exponents with scientific notation.**

In scientific notation, a number is written with the decimal point after the first nonzero digit and multiplied by a power of 10.

This is often a simpler way to express very large or very small numbers.

---

**科学记数法**

科学记数法是一个数当它表示为形式

\[ a \times 10^n \]

其中 \(1 \leq |a| < 10\) 且 \(n\) 是一个整数。

---

**Use the rules for exponents with scientific notation.**

**Converting to Scientific Notation**

1. **Step 1 Position the decimal point.** Place a caret, \(\ast\), to the right of the first nonzero digit, where the decimal point will be placed.

2. **Step 2 Determine the numeral for the exponent.** Count the number of digits from the decimal point to the caret. This number gives the absolute value of the exponent on 10.

3. **Step 3 Determine the sign for the exponent.** Decide whether multiplying by \(10^n\) should make the result of **Step 1** greater or less. The exponent should be positive to make the result greater; it should be negative to make the result less.
Writing Numbers in Scientific Notation

Write the number in scientific notation.

29,800,000

Solution:

**Step 1** Place a caret to the right of the first nonzero digit (2) to mark the new location of the decimal point.

**Step 2** Count from the decimal point 7 places, which is understood to be after the caret.

29,800,000 = 2.98,000,000 - Decimal point moves 7 places to the left

**Step 3** Since 2.98 is to be made greater, the exponent on 10 is positive.

29,800,000 = 2.98 × 10^7

Write the number in scientific notation.

0.0000000503

Solution:

**Step 1** Place a caret to the right of the first nonzero digit (5) to mark the new location of the decimal point.

**Step 2** Count from the decimal point 8 places, which is understood to be after the caret.

0.0000000503 = 0.00000005.03 - Decimal point moves 7 places to the left

**Step 3** Since 5.03 is to be made less, the exponent 10 is negative.

0.0000000503 = 5.03 × 10^-8

When converting from scientific notation to standard notation, use the exponent to determine the number of places and the direction in which to move the decimal point.

2.51 ×10^3

2.51 ×10^3 = 2510

Move the decimal 3 places to the right.

−6.8 ×10^-4

−6.8 ×10^-4 = −0.000068

Move the decimal 4 places to the left.

The distance to the sun is 9.3 × 10^7 mi. How long would it take a rocket traveling at 3.2 × 10^3 mph to reach the sun?

\[ d = rt, \quad \text{so} \quad t = \frac{d}{r} \]

\[ t = \frac{9.3 \times 10^7}{3.2 \times 10^3} = \frac{9.3}{3.2} \times 10^{7-3} = 2.9 \times 10^4 \]

It would take approximately 2.9 × 10^4 hours.
Adding and Subtracting Polynomials

Objectives
1. Know the basic definitions for polynomials.
2. Add and subtract polynomials.

A term is a number (constant), a variable, or the product or quotient of a number and one or more variables raised to powers.

The number in the product is called the numerical coefficient, or just the coefficient.

A term or a sum of two or more terms is an algebraic expression.

The simplest kind of algebraic expression is a polynomial.

A polynomial containing only the variable \( x \) is called a polynomial in \( x \).

A polynomial in one variable is written in descending powers of the variable if the exponents on the variable decrease from left to right.

When written in descending powers of the variable, the greatest-degree term is written first and is called the leading term of the polynomial. Its coefficient is the leading coefficient.

If a polynomial in a single variable is written in descending powers of that variable, the degree of the polynomial will be the degree of the leading term.

Some polynomials with a specific number of terms are so common that they are given special names.

<table>
<thead>
<tr>
<th>Type of Polynomial</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monomial</td>
<td>( 5x, 7n^2 )</td>
</tr>
<tr>
<td>Binomial</td>
<td>( 3x^2 - 6, 11y + 8, 5x^2 + 3a )</td>
</tr>
<tr>
<td>Trinomial</td>
<td>( x^2 + 11y + 6, 8p^2 - 7p + 2m, -3 + 2k^2 + 9z^4 )</td>
</tr>
<tr>
<td>None of these</td>
<td>( p^2 - 5p^3 + 2p - 5, -9z^4 + 5z^2 + 2m^3 + 11n^2 - 7r )</td>
</tr>
</tbody>
</table>

The number 0 has no degree, since 0 times a variable to any power is 0.
Identify each polynomial as a monomial, binomial, trinomial, or none of these. Also, give the degree.

- \( a^4b^2 - ab^6 \)
  Solution:
  Binomial of degree of 7
- \(-100\)
  Monomial of degree of 0

### Classifying Polynomials

#### Objective 2
Add and subtract polynomials.

#### Adding Polynomials
To add two polynomials, combine like terms.

#### Subtracting Polynomials
Subtract:
\((p^4 + p^3 + 5) - (3p^4 + 5p^3 + 2)\)

Solution:
Change every sign in the second polynomial and add.
\[ p^4 + p^3 + 5 - 3p^4 - 5p^3 - 2 \]
\[ = p^4 - 3p^4 + p^3 - 5p^3 + 5 - 2 \]
\[ = -2p^4 - 4p^3 + 3 \]

- \( 2k^3 - 3k^2 - 2k + 5 \)
- \( 4k^3 + 6k^2 - 5k + 8 \)
- \( -2k^3 - 9k^2 + 3k - 3 \)

To subtract vertically, write the first polynomial above the second, lining up like terms in columns. Change all the signs in the second polynomial and add.
Recognize and evaluate polynomial functions.

**Polyomial Function**

A polynomial function of degree \( n \) is defined by

\[
f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0,
\]

for real numbers \( a_n, a_{n-1}, \ldots, a_1, \) and \( a_0, \) where \( a_n \neq 0 \) and \( n \) is a whole number.

---

**CLASSROOM EXAMPLE 1** Evaluating Polynomial Functions

Let \( f(x) = -x^2 + 5x - 11. \) Find \( f(-4) \)

**Solution:**

\[
f(-4) = -(-4)^2 + 5(-4) - 11
= -16 - 20 - 11
= -47
\]

---

**CLASSROOM EXAMPLE 2** Using a Polynomial Model to Approximate Data

The number of students enrolled in public schools (grades pre-K-12) in the United States during the years 1990 through 2006 can be modeled by the polynomial function defined by

\[
P(x) = -0.01774x^2 + 0.7871x + 41.26,
\]

where \( x = 0 \) corresponds to the year 1990, \( x = 1 \) corresponds to 1991, and so on, and \( P(x) \) is in millions. Use this function to approximate the number of public school students in 2000.

\[
P(10) = -0.01774(10)^2 + 0.7871(10) + 41.26
= -17.74 + 7.871 + 41.26
= 47.4 \text{ million students}
\]

---

Add and subtract polynomial functions.

The operations of addition, subtraction, multiplication, and division are also defined for functions.

For example, businesses use the equation “profit equals revenue minus cost,” written in function notation as

\[
P(x) = R(x) - C(x)
\]

where \( x \) is the number of items produced and sold.

---

Add and subtract polynomial functions.

If \( f(x) \) and \( g(x) \) define functions, then

\[
(f + g)(x) = f(x) + g(x) \quad \text{Sum function}
\]

and

\[
(f - g)(x) = f(x) - g(x) \quad \text{Difference function}
\]

In each case, the domain of the new function is the intersection of the domains of \( f(x) \) and \( g(x) \).
For \( f(x) = 3x^2 + 8x - 6 \) and \( g(x) = -4x^2 + 4x - 8 \), find each of the following.

\[
(f + g)(x) = f(x) + g(x) = \frac{\frac{\frac{8}{8}}{8}}{8} x^2 + \frac{\frac{1}{8}}{8} - \frac{\frac{1}{8}}{8} + \frac{\frac{1}{8}}{8} + 8 = -x^2 + 12x - 14
\]

\[
(f - g)(x) = f(x) - g(x) = \frac{\frac{\frac{1}{8}}{8}}{8} x^2 + \frac{\frac{1}{8}}{8} + \frac{\frac{1}{8}}{8} - \frac{\frac{1}{8}}{8} + 8 = 3x + 4 + 8
\]

For \( f(x) = 18x^2 - 24x \) and \( g(x) = 3x \), find each of the following.

\[
(f + g)(x) \quad \text{and} \quad (f + g)(-1)
\]

\[
(f - g)(x) \quad \text{and} \quad (f - g)(1)
\]

\[
(f + g)(x) = f(x) + g(x) = 18x^2 - 24x + 3x = 18x^2 - 21x
\]

\[
(f + g)(-1) = f(-1) + g(-1) = [18(-1)^2 - 24(-1)] + 3(-1) = [18 + 24] - 3 = 39
\]

\[
(f - g)(x) = f(x) - g(x) = 18x^2 - 24x - 3x = 18x^2 - 27x
\]

\[
(f - g)(1) = f(1) - g(1) = [18(1)^2 - 24(1)] - 3 = 18 - 24 - 3 = -9
\]

The simplest polynomial function is the identity function, defined by \( f(x) = x \) and graphed below. This function pairs each real number with itself.

The domain (set of \( x \)-values) is all real numbers, \((-\infty, \infty)\). The range (set of \( y \)-values) is also \((-\infty, \infty)\).
Another polynomial function, defined by \( f(x) = x^2 \) and graphed below, is the \textit{squaring function}. For this function, every real number is paired with its square. The graph of the squaring function is a \textit{parabola}.

The domain is all real numbers, \((-\infty, \infty)\).

The range is \([0, \infty)\).

The \textit{cubing function} is defined by \( f(x) = x^3 \) and graphed below. This function pairs every real number with its cube.

The domain and range are both \((-\infty, \infty)\).

Graph basic polynomial functions.

### CLASSROOM EXAMPLE 7

Graph \( f(x) = -2x^2 \). Give the domain and range.

\[
\begin{array}{c|c}
 x & f(x) = -2x^2 \\
 \hline
-2 & -8 \\
-1 & -2 \\
0 & 0 \\
1 & -2 \\
2 & -8 \\
\end{array}
\]

The domain is \((-\infty, \infty)\).

The range is \((-\infty, 0]\).

Graph basic polynomial functions.

### CLASSROOM EXAMPLE 7

Graphing Variations of Polynomial Functions

Graph \( f(x) = -2x^2 \). Give the domain and range.

Solution:

\[
\begin{array}{c|c}
 x & f(x) = -2x^2 \\
 \hline
-2 & -8 \\
-1 & -2 \\
0 & 0 \\
1 & -2 \\
2 & -8 \\
\end{array}
\]

The domain is \((-\infty, \infty)\).

The range is \((-\infty, 0]\).
### Multiplying Polynomials

#### Objectives
1. Multiply terms.
2. Multiply any two polynomials.
4. Find the product of the sum and difference of two terms.
5. Find the square of a binomial.

#### Example 1: Multiplying Monomials

**Find the product.**

\[
8k^3y(9k^4y)
\]

**Solution:**

\[
= (8)(9)k^3 \cdot k^4 \cdot y \cdot y^1
\]

\[
= 72k^{3+4}y^{1+1}
\]

\[
= 72k^7y^2
\]

#### Example 2: Multiplying Polynomials

**Find the product.**

\[
-2r(9r - 5)
\]

**Solution:**

\[
= -2r(9r) - 2r(-5)
\]

\[
= -18r^2 + 10r
\]

#### Example 2 (cont'd): Multiplying Polynomials

**Find the product.**

\[
(4x - 3y)(3x - y)
\]

**Solution:**

\[
= 4x(3x) + 4x(-y) - 3y(3x) - 3y(-y)
\]

\[
= 12x^2 - 4xy - 9xy + 3y^2
\]

\[
= 12x^2 - 13xy + 3y^2
\]

Combine like terms.

#### Example 3: Multiplying Polynomials Vertically

**Find the product.**

\[
(2k - 5m)(3k + 2m)
\]

**Solution:**

\[
= (2k)(3k) + (2k)(2m) - 5m(3k) - 5m(2m)
\]

\[
= 6k^2 + 4km - 15km - 10m^2
\]

\[
= 6k^2 - 11km - 10m^2
\]
Find the product.

\((5a^3 - 6a^2 + 2a - 3)(2a - 5)\)

Solution:

\[
\begin{array}{c}
5a^3 - 6a^2 + 2a - 3 \\
2a - 5 \\
\hline
-25a^3 + 30a^2 - 10a + 15 \\
10a^4 - 12a^3 + 4a^2 - 6a \\
\hline
10a^4 - 37a^3 + 34a^2 - 16a + 15
\end{array}
\]

Combine like terms.

Multiply binomials.

When working with polynomials, the products of two binomials occurs repeatedly. There is a shortcut method for finding these products.

First Terms
Outer Terms
Inner Terms
Last Terms

The FOIL method is an extension of the distributive property, and the acronym "FOIL" applies only to multiplying two binomials.

Find each product.

\((m + 5)(m - 5)\) = \(m^2 - 25\)

\((x - 4)(x + 4)\) = \(x^2 - 16\)

\(4y^2y + 7(x - 7)\) = \(4y^4 - 49\)

Find the product of the sum and difference of two terms.

Product of the Sum and Difference of Two Terms

The product of the sum and difference of the two terms \(x\) and \(y\) is the difference of the squares of the terms.

\((x + y)(x - y) = x^2 - y^2\)
Square of a Binomial

The square of a binomial is the sum of the square of the first term, twice the product of the two terms, and the square of the last term.

\[(x + y)^2 = x^2 + 2xy + y^2\]

\[(x - y)^2 = x^2 - 2xy + y^2\]

Find the square of a binomial.

Find each product.

\[(t + 9)^2 = t^2 + 2 \cdot t \cdot 9 + 9^2 = t^2 + 18t + 81\]

\[(2m + 5)^2 = (2m)^2 + 2(2m)(5) + 5^2 = 4m^2 + 20m + 25\]

\[(3k - 2n)^2 = (3k)^2 - 2(3k)(2n) + (2n)^2 = 9k^2 - 12kn + 4n^2\]

CLASSROOM EXAMPLE 6

Squaring Binomials

Find each product.

\[\[(x - y) + z][(x - y) - z]\]

\[= (x - y)^2 - z^2\]

\[= x^2 - 2(x)(y) + y^2 - z^2\]

\[= x^2 - 2xy + y^2 - z^2\]

\[(p + 2q)^2 = (p + 2q)(p + 2q)\]

\[= p^2 + 4pq + 4q^2\]

\[= p^2 + 8pq + 4q^2\]

\[= p^2 + 12pq + 6q^2\]

\[(x + 2)^2 = (x + 2)(x + 2)\]

\[= (x^2 + 4x + 4)(x^2 + 4x + 4)\]

\[= x^4 + 4x^3 + 4x^2 + 16x^2 + 16x + 16x + 16\]

\[= x^4 + 8x^3 + 24x^2 + 32x + 16\]

Multiply polynomial functions.

Multiply polynomial functions.

CLASSROOM EXAMPLE 7

Multiplying More Complicated Binomials

Find each product.

\[f(x) = 3x + 1\] and \(g(x) = 2x - 5\), find \((f \cdot g)(x)\) and \((f \cdot g)(2)\).

\[(f \cdot g)(x) = f(x) \cdot g(x)\].

\[= (3x + 1)(2x - 5)\]

\[= 6x^2 - 15x + 2x - 5\]

\[= 6x^2 - 13x - 5\]

\[(f \cdot g)(2) = 6(2)^2 - 13(2) - 5\]

\[= 24 - 26 - 5\]

\[= -7\]
# Dividing Polynomials

## Objectives

1. Divide a polynomial by a monomial.
2. Divide a polynomial by a polynomial of two or more terms.
3. Divide polynomial functions.

## Dividing a Polynomial by a Monomial

To divide a polynomial by a monomial, divide each term in the polynomial by the monomial, and then write each quotient in lowest terms.

### Example 1

Divide.

\[
\frac{10x^2 - 25x + 35}{5}
\]

**Solution:**

\[
\frac{10x^2}{5} - \frac{25x}{5} + \frac{35}{5} = 2x^2 - 5x + 7
\]

Check this answer by multiplying it by the divisor, 5.

\[
5(2x^2 - 5x + 7) = 10x^2 - 25x + 35
\]

### Example 2

Divide.

\[
\frac{6a^2b^4 - 9a^2b^3 + 4a^2b^4}{a^2b^4}
\]

**Solution:**

\[
\frac{6a^2b^4}{a^2b^4} - \frac{9a^2b^3}{a^2b^4} + \frac{4a^2b^4}{a^2b^4} = 6 - \frac{9a}{b} + 4a
\]

### Example 3

Divide a polynomial by a polynomial of two or more terms.

**Solution:**

Write the problem as if dividing whole numbers, make sure that both polynomials are written in descending powers of the variables.

\[
\frac{2k^2 + 17k + 30}{k + 6}
\]

Divide the first term of \(2k^2\) by the first term of \(k + 6\). Write the result above the division line.
Multiply and write the result below.

\[
\frac{2k + 5}{k + 6} \cdot \frac{2k^2 + 17k + 30}{2k^2 + 12k}
\]

Subtract. Do this mentally by changing the signs on \(2k^2 + 12k\) and adding.

\[
\frac{5k + 30}{5k + 30}
\]

You can check the result, \(2k + 5\), by multiplying \(k + 6\) and \(2k + 5\).
Divide polynomial functions.

**Dividing Functions**

If \( f(x) \) and \( g(x) \) define functions, then

\[
\left( \frac{f}{g} \right)(x) = \frac{f(x)}{g(x)}
\]

Quotient function

The domain of the quotient function is the intersection of the domains of \( f(x) \) and \( g(x) \), excluding any values of \( x \) for which \( g(x) = 0 \).

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**CLASSROOM EXAMPLE 6**

**Dividing Polynomial Functions**

For \( f(x) = 2x^2 + 17x + 30 \) and \( g(x) = 2x + 5 \), find \( \left( \frac{f}{g} \right)(x) \) and \( \left( \frac{f}{g} \right)(-1) \).

Solution:

From previous Example 2, we conclude that \( (f/g)(x) = x + 6 \), provided the denominator \( 2x + 5 \), is not equal to zero.

\[
\left( \frac{f}{g} \right)(x) = \frac{f(x)}{g(x)} = \frac{2x^2 + 17x + 30}{2x + 5} = x + 6
\]

\[
\left( \frac{f}{g} \right)(-1) = -1 + 6 = 5
\]