7.3 Complex Fractions

Objectives

1. Simplify complex fractions by simplifying the numerator and denominator (Method 1).
2. Simplify complex fractions by multiplying by a common denominator (Method 2).
3. Compare the two methods of simplifying complex fractions.
4. Simplify rational expressions with negative exponents.

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Complex Fractions

A **complex fraction** is a quotient having a fraction in the numerator, denominator, or both.

$$\frac{1+\frac{1}{x}}{2}, \quad \frac{\frac{4}{y}}{6-\frac{3}{y}}, \quad \text{and} \quad \frac{\frac{m^2-9}{m+1}}{\frac{m+3}{m^2-1}}.$$

Objective 1

Simplify complex fractions by simplifying the numerator and denominator (Method 1).

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Simplifying a Complex Fraction: Method 1

Step 1 Simplify the numerator and denominator separately.

Step 2 Divide by multiplying the numerator by the reciprocal of the denominator.

Step 3 Simplify the resulting fraction if possible.

CLASSROOM EXAMPLE 1 **Simplifying Complex Fractions (Method 1)**

Use Method 1 to simplify the complex fraction.

$$\frac{\frac{y+2}{y}}{\frac{y-2}{3y}}$$

Both the numerator and denominator are already simplified.

Solution:

$$=\frac{y+2}{y}\div\frac{y-2}{3y}$$ **Write as a division problem.**

$$=\frac{y+2}{y}\cdot\frac{3y}{y-2}$$ **Multiply by the reciprocal.**

$$=\frac{3(y+2)}{y-2}$$ **Multiply.**

CLASSROOM EXAMPLE 1 **Simplifying Complex Fractions (Method 1) (cont'd)**

Use Method 1 to simplify the complex fraction.

$$\frac{4-\frac{3}{x}}{5-\frac{1}{x}}$$

Simplify the numerator and denominator.

Solution:

$$=\frac{4\left(\frac{x}{x}\right)-\frac{3}{x}}{5\left(\frac{x}{x}\right)-\frac{1}{x}}$$

$$=\frac{\frac{4x}{x}-\frac{3}{x}}{\frac{5x}{x}-\frac{1}{x}}$$

$$=\frac{\frac{4x-3}{x}}{\frac{5x-1}{x}}$$

$$=\frac{4x-3}{x}\div\frac{5x-1}{x}$$

$$=\frac{4x-3}{x}\cdot\frac{x}{5x-1} = \frac{4x-3}{5x-1}$$

Simplify complex fractions by multiplying by a common denominator (Method 2).

Simplifying a Complex Fraction: Method 2

Step 1 Multiply the numerator and denominator of the complex fraction by the least common denominator of the fractions in the numerator and the fractions in the denominator of the complex fraction.

Step 2 Simplify the resulting fraction if possible.

CLASSROOM EXAMPLE 2 **Simplifying Complex Fractions (Method 2)**

Use Method 2 to simplify the complex fraction.

$$\frac{4-\frac{3}{x}}{5-\frac{1}{x}}$$

The LCD is *x*. Multiply the numerator and denominator by *x*.

Solution:

$$=\frac{\left(4-\frac{3}{x}\right)\cdot x}{\left(5-\frac{1}{x}\right)\cdot x} \qquad =\frac{4\cdot x-\frac{3}{x}\cdot x}{5\cdot x-\frac{1}{x}\cdot x} \qquad =\frac{4x-3}{5x-1}$$

CLASSROOM EXAMPLE 2 **Simplifying Complex Fractions (Method 2) (cont'd)**

Use Method 2 to simplify the complex fraction.

$$\frac{3y+\frac{4}{y+1}}{2y-\frac{3}{y}}$$

Multiply the numerator and denominator by the LCD *y*(*y* + 1).

Solution:

$$=\frac{\left(3y+\frac{4}{y+1}\right)\cdot y(y+1)}{\left(2y-\frac{3}{y}\right)\cdot y(y+1)} \qquad =\frac{3y[y(y+1)]+\frac{4}{y+1}\cdot y(y+1)}{2y[y(y+1)]-\frac{3}{y}\cdot y(y+1)}$$

$$=\frac{3y^2(y+1)+4y}{2y^2(y+1)-3(y+1)} \qquad =\frac{3y^3+3y^2+4y}{2y^3+2y^2-3y-3}$$

Objective 3

Compare the two methods of simplifying complex fractions.

CLASSROOM EXAMPLE 3 **Simplifying Complex Fractions (Both Methods)**

Simplify the complex fraction by both methods.

$$\frac{\frac{5}{y+2}}{\frac{-3}{y^2-4}}$$

Solution:

Method 1

$$=\frac{\frac{5}{y+2}}{\frac{-3}{(y-2)(y+2)}}$$

$$=\frac{5}{y+2}\div\frac{-3}{(y+2)(y-2)}$$

$$=\frac{5}{y+2}\cdot\frac{(y+2)(y-2)}{-3}$$

$$=\frac{5(y-2)}{-3}$$

Method 2

$$=\frac{\frac{5}{y+2}}{\frac{-3}{(y-2)(y+2)}}$$

$$=\frac{\frac{5}{y+2}\cdot(y+2)(y-2)}{\frac{-3}{(y-2)(y+2)}\cdot(y+2)(y-2)}$$

$$=\frac{5(y-2)}{-3}$$

CLASSROOM EXAMPLE 3 **Simplifying Complex Fractions (Both Methods) (cont'd)**

Simplify the complex fraction by both methods.

$$\frac{\frac{1}{a}-\frac{1}{b}}{\frac{1}{a^2}-\frac{1}{b^2}}$$

Solution: **Method 1**

$$=\frac{\frac{b}{ab}-\frac{a}{ab}}{\frac{b^2}{a^2b^2}-\frac{a^2}{a^2b^2}}$$

LCD = *ab*

LCD = *a*²*b*²

$$=\frac{\frac{b-a}{ab}}{\frac{b^2-a^2}{a^2b^2}}$$

$$=\frac{\frac{b-a}{ab}}{\frac{(b+a)(b-a)}{a^2b^2}}$$

$$=\frac{b-a}{ab}\div\frac{(b+a)(b-a)}{a^2b^2}$$

$$=\frac{b-a}{ab}\cdot\frac{a^2b^2}{(b+a)(b-a)}$$

$$=\frac{ab}{b+a}$$

CLASSROOM EXAMPLE 3 — **Simplifying Complex Fractions (Both Methods) (cont'd)**

Simplify the complex fraction by both methods.

$$\frac{\frac{1}{a}-\frac{1}{b}}{\frac{1}{a^2}-\frac{1}{b^2}}$$

Solution: Method 2

LCD of the numerator and denominator is a^2b^2.

$$=\frac{\left(\frac{1}{a}-\frac{1}{b}\right)\cdot a^2b^2}{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\cdot a^2b^2} \qquad =\frac{ab^2-a^2b}{b^2-a^2}$$

$$=\frac{ab(b-a)}{(b+a)(b-a)} \qquad =\frac{ab}{b+a}$$

CLASSROOM EXAMPLE 4 — **Simplifying Rational Expressions with Negative Exponents**

Simplify the expression, using only positive exponents in the answer.

$$\frac{a^{-2}+b^{-1}}{a^{-1}-5b^{-3}}$$

Solution:

$$=\frac{\frac{1}{a^2}+\frac{1}{b}}{\frac{1}{a}-\frac{5}{b^3}}$$

LCD = a^2b^3

$$=\frac{a^2b^3\left(\frac{1}{a^2}+\frac{1}{b}\right)}{a^2b^3\left(\frac{1}{a}-\frac{5}{b^3}\right)}$$

$$=\frac{a^2b^3\cdot\frac{1}{a^2}+a^2b^3\cdot\frac{1}{b}}{a^2b^3\cdot\frac{1}{a}-a^2b^3\cdot\frac{5}{b^3}} \qquad =\frac{b^3+a^2b^2}{ab^3-5a^2}$$

CLASSROOM EXAMPLE 4 — **Simplifying Rational Expressions with Negative Exponents (cont'd)**

Simplify the expression, using only positive exponents in the answer.

$$\frac{x^{-3}+2y^{-1}}{y+2x^3}$$

Solution:

$$=\frac{\frac{1}{x^3}+\frac{2}{y}}{y+2x^3}$$

Write with positive exponents.

$$=\frac{\frac{1\cdot y}{x^3\cdot y}+\frac{2\cdot x^3}{y\cdot x^3}}{y+2x^3}$$

LCD = x^3y

$$=\frac{\frac{y+2x^3}{x^3y}}{\frac{y+2x^3}{1}}$$

$$=\frac{y+2x^3}{x^3y}\div\frac{y+2x^3}{1}$$

$$=\frac{y+2x^3}{x^3y}\cdot\frac{1}{y+2x^3}$$

$$=\frac{1}{x^3y}$$