Objectives

1. Simplify complex fractions by simplifying the numerator and denominator (Method 1).
2. Simplify complex fractions by multiplying by a common denominator (Method 2).
3. Compare the two methods of simplifying complex fractions.
4. Simplify rational expressions with negative exponents.

Complex Fractions

A complex fraction is a quotient having a fraction in the numerator, denominator, or both.

\[
\frac{1 + \frac{1}{x}}{2}, \quad \frac{4}{y}, \quad \frac{m^2 - 9}{6 - \frac{3}{y}}, \quad \text{and} \quad \frac{m + 1}{m^3 - 1}
\]

Objective 1

Simplify complex fractions by simplifying the numerator and denominator (Method 1).

Simplifying Complex Fractions (Method 1)

Use Method 1 to simplify the complex fraction.

\[
\frac{y + 2}{y} \div \frac{y - 2}{3y}
\]

Solution:

\[
= \frac{y + 2}{y} \div \frac{y - 2}{3y} = \frac{y + 2}{y} \cdot \frac{3y}{y - 2} = \frac{3(y + 2)}{y - 2}
\]

Both the numerator and denominator are already simplified.

Write as a division problem.

Multiply by the reciprocal.

Multiply.

Simplifying Complex Fractions (Method 1) (cont’d)

Use Method 1 to simplify the complex fraction.

\[
\frac{4 - \frac{3}{x}}{5 - \frac{1}{x}}
\]

Solution:

\[
= \frac{4}{x} \div \frac{3}{x} = \frac{4}{x} \cdot \frac{x}{3} = \frac{4}{3} = \frac{4x - 3}{3x - 1}
\]

Simplify the numerator and denominator.

\[
= \frac{4x - 3}{3x - 1} = \frac{4x - 3}{x} \div \frac{x}{x} \div \frac{x}{5x - 1} = \frac{4x - 3}{x} \div \frac{x}{5x - 1} = \frac{4x - 3}{x} \div \frac{x}{5x - 1} = \frac{4x - 3}{5x - 1}
\]
Simplify complex fractions by multiplying by a common denominator (Method 2).

**Simplifying a Complex Fraction: Method 2**

**Step 1** Multiply the numerator and denominator of the complex fraction by the least common denominator of the fractions in the numerator and the fractions in the denominator of the complex fraction.

**Step 2** Simplify the resulting fraction if possible.

**CLASSROOM EXAMPLE 2**

Use Method 2 to simplify the complex fraction.

\[
\frac{\frac{3}{x}}{\frac{1}{x}}
\]

**Solution:**

\[
\frac{\frac{4 - \frac{3}{x}}{5 - \frac{1}{x}} \cdot x}{\frac{4 \cdot \frac{3}{x}}{5 \cdot \frac{1}{x}}}
\]

\[
\frac{4x - 3}{5x - 1}
\]

**Objective 3**

Compare the two methods of simplifying complex fractions.

**CLASSROOM EXAMPLE 3**

Simplify the complex fraction by both methods.

**Solution:**

**Method 1**

\[
\frac{5 \frac{y + 2}{y^2 - 4}}{-3}
\]

**Method 2**

\[
\frac{\frac{5}{y + 2}}{-3}
\]

**Classroom Example 3 (cont'd)**

Simplify the complex fraction by both methods.

**Solution:**

**Method 1**

\[
\frac{\frac{1}{a} - \frac{b}{b^2}}{\frac{1}{a^2} + \frac{b}{b^2}}
\]

**Method 2**

\[
\frac{\frac{b - a}{ab}}{\frac{(b + a)(b - a)}{a^2b^2}}
\]
CLASSROOM EXAMPLE 3

Simplifying Complex Fractions (Both Methods) (cont’d)

Simplify the complex fraction by both methods.

\[
\frac{1}{a} \cdot \frac{b}{b} = \frac{1}{a^2 - b^2} \quad \text{Solution: Method 2}
\]

LCD of the numerator and denominator is \(a^2b^2\).

\[
\frac{\left( \frac{1}{a} \cdot \frac{1}{b} \right)}{\left( \frac{1}{a^2} - \frac{1}{b^2} \right) \cdot a^2b^2} = \frac{ab - a^2b}{b^2 - a^2}
\]

\[
= \frac{ab(b - a)}{(b + a)(b - a)} \quad = \frac{ab}{b + a}
\]

CLASSROOM EXAMPLE 4

Simplifying Rational Expressions with Negative Exponents (cont’d)

Simplify the expression, using only positive exponents in the answer.

\[
\frac{a^2 + b^{-1}}{a^2 - 5b^{-3}}
\]

Solution:

\[
\frac{1}{a^2 + b} - \frac{5}{a} \cdot b^{-2}
\]

\[
= \frac{1}{a^2 + b} - \frac{5}{a} \cdot b^{-2}
\]

\[
= \frac{1}{a^2 + b} - \frac{5}{a} \cdot b^{-2}
\]

\[
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CLASSROOM EXAMPLE 4

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\]

\[
= \frac{ab}{b + a}
\]