7.6 Variation

Objectives

1 Write an equation expressing direct variation.

2 Find the constant of variation, and solve direct variation problems.

3 Solve inverse variation problems.

4 Solve joint variation problems.

5 Solve combined variation problems.

PEARSON

Write an equation expressing direct variation.

Direct Variation

y* varies directly as *x if there exists a real number k such that

$$y = kx.$$

y is said to be **proportional to** x. The number k is called the **constant of variation.**

In direct variation, for k > 0, as the value of x increases, the value of y also increases. Similarly, as x decreases, y decreases.

CLASSROOM EXAMPLE 1 **Finding the Constant of Variation and the Variation Equation**

If 7 kg of steak cost $45.50, how much will 1 kg of steak cost?

Solution:

Let C represent the cost of p kilograms of steak. C varies directly as p, so $C = kp$.

Here k represents the cost of one kilogram of steak. Since $C = 45.50$ when $p = 7$,

$$45.50 = k \cdot 7.$$

$$k = \frac{45.50}{7}$$

$$k = 6.50$$

One kilogram of steak costs $6.50, and C and p are related by $C = 6.50p$.

CLASSROOM EXAMPLE 2 **Solving a Direct Variation Problem**

It costs $52 to use 800 kilowatt-hours of electricity. How much will 650 kilowatt-hours cost?

Solution:

Let c represent the cost of using h kilowatt-hours. Use $c = kh$ with $c = 52$ and $h = 800$ to find k.

$$c = kh$$

$$52 = k(800)$$

$$\frac{52}{800} = k$$

$$\frac{13}{200} = k$$

CLASSROOM EXAMPLE 2 **Solving a Direct Variation Problem (cont'd)**

So $c = \frac{13}{200}h.$

Let $h = 650$. Find c.

$$c = \frac{13}{200}(650)$$

$$c = 42.25$$

Thus, 650 kilowatt-hours costs $42.25.

Find the constant of variation, and solve direct variation problems.

Solving a Variation Problem

Step 1 Write the variation equation.

Step 2 Substitute the initial values and solve for k.

Step 3 Rewrite the variation equation with the value of k from ***Step 2.***

Step 4 Substitute the remaining values, solve for the unknown, and find the required answer.

Find the constant of variation, and solve direct variation problems.

Direct Variation as a Power

y* varies directly as the *n*th power of *x if there exists a real number *k* such that

$$y = kx^n.$$

CLASSROOM EXAMPLE 3 **Solving a Direct Variation Problem**

Suppose y varies directly as the cube of x, and $y = 24$ when $x = 2$. Find y when $x = 4$.

Solution:

Step 1 y varies directly as the cube of x, so $y = kx^3$.

Step 2 Find the value of k.

$y = 24$ when $x = 2$, so

$$24 = k(2)^3$$
$$24 = k(8)$$
$$3 = k$$

Step 3 Thus, $y = 3x^3$.

Step 4 When $x = 4$, $y = 3(4)^3 = 3(64) = 192$.

Solve inverse variation problems.

Inverse Variation

y* varies inversely as *x if there exists a real number *k* such that

$$y = \frac{k}{x}.$$

Also, ***y* varies inversely as the *n*th power of *x*** if there exists a real number *k* such that

$$y = \frac{k}{x^n}.$$

With inverse variation, where k > 0, as one variable increases, the other variable decreases.

CLASSROOM EXAMPLE 4 **Solving an Inverse Variation Problem**

The current in a simple electrical circuit varies inversely as the resistance. If the current is 80 amps when the resistance is 10 ohms, find the current if the resistance is 16 ohms.

Solution:

Let C represent the current, and R the resistance.
C varies inversely as R, so

$$C = \frac{k}{R}, \quad \text{for some constant } k.$$

Since $C = 80$ when $R = 10$,

$$80 = \frac{k}{10}$$
$$800 = k.$$

CLASSROOM EXAMPLE 4 **Solving an Inverse Variation Problem (cont'd)**

Thus $C = \frac{800}{R}.$

When $R = 16,$

$$C = \frac{800}{16} = 50.$$

The current is 50 ampere.

CLASSROOM EXAMPLE 5 **Solve an Inverse Variation Problem**

Suppose p varies inversely as the cube of q and $p = 100$ when $q = 3$. Find p, given that $q = 5$.

Solution:

p varies inversely as the cube of q, so $p = \frac{k}{q^3}.$

$p = 100$ when $q = 3$, so $100 = \frac{k}{3^3}$

$$2700 = k$$

Thus, $p = \frac{2700}{q^3}.$ When $q = 5,$

$$p = \frac{2700}{5^3} = \frac{2700}{125} = \frac{108}{5}.$$

Objective 4

Solve joint variation problems.

Solve joint variation problems.

Joint Variation

y* varies jointly as *x* and *z if there exists a real number *k* such that ***y* = *kxz*.**

CAUTION: Note that ***and*** in the expression "*y* varies directly as *x* and *z*" translates as the product *y* = *kxz*. The word ***and*** does not indicate addition here.

CLASSROOM EXAMPLE 6 **Solving a Joint Variation Problem**

If *x* varies jointly as *y* and z^2, and *x* = 231 when *y* = 3 and *z* = 2, find *x* when *y* = 5 and *z* = 4.

Solution:

x varies jointly as *y* and z^2, so $x = kyz^2$.

x = 231 when *y* = 3 and *z* = 2, so $231 = k(3)(2)^2$

$$77 = k(4)$$

$$k = \frac{77}{4}.$$

CLASSROOM EXAMPLE 6 **Solving a Joint Variation Problem (cont'd)**

Thus, $x = \frac{77}{4} yz^2$.

When *y* = 5 and *z* = 4,

$$x = \frac{77}{4}(5)(4)^2 = 77(5)(4) = 1540.$$

CLASSROOM EXAMPLE 7 **Solving a Combined Variation Problem**

Suppose *z* varies jointly as *x* and y^2 and inversely as *w*. Also, $z = \frac{3}{8}$ when *x* = 2, *y* = 3, and *w* = 12. Find *z*, when *x* = 4, *y* = 1, and *w* = 6.

Solution:

z varies jointly as *x* and y^2 and inversely as *w*, so

$$z = \frac{kxy^2}{w}.$$

CLASSROOM EXAMPLE 7 **Solving a Combined Variation Problem (cont'd)**

$z = \frac{3}{8}$ when *x* = 2, *y* = 3 and *w* = 12, so $\frac{3}{8} = \frac{k(2)(3)^2}{12}$

$$\frac{3}{8} = \frac{k(3)}{2}$$

$$k = \frac{3}{8} \cdot \frac{2}{3} = \frac{1}{4}$$

Thus, $z = \frac{\frac{1}{4}xy^2}{w} = \frac{xy^2}{4w}$.

When *x* = 4, *y* = 1, and *w* = 6, $z = \frac{4(1)^2}{4(6)} = \frac{1}{6}$.