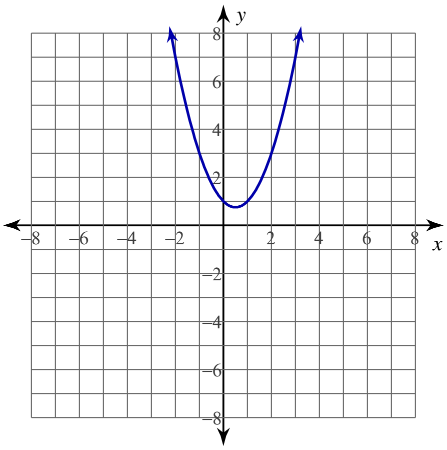
Name________________________________

Average Rates of Change

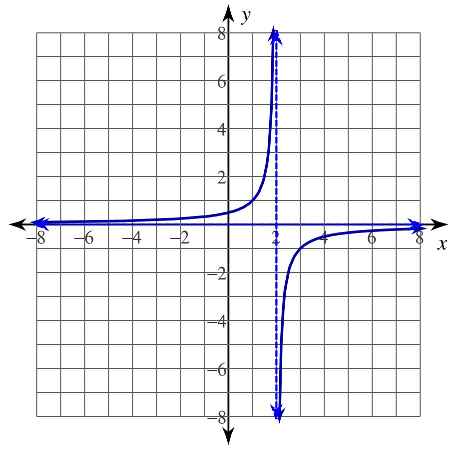
Date________________ Period____

For each problem, find the average rate of change of the function over the given interval.

1) $y = x^2 - x + 1;\quad [0, 3]$

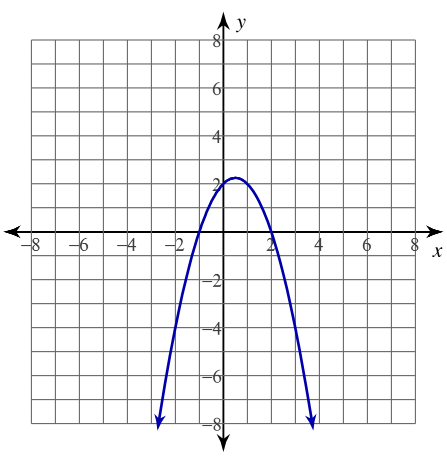


2) $y = -\dfrac{1}{x - 2};\quad [-3, -2]$

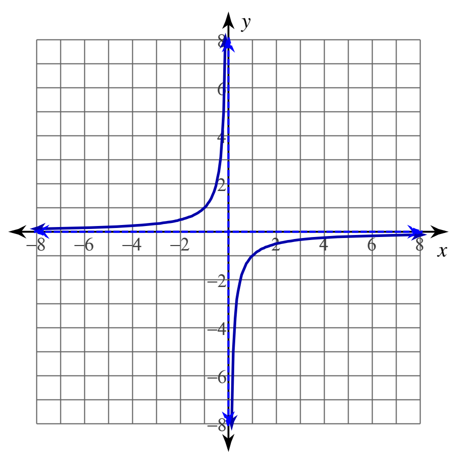


For each problem, find the equation of the secant line that intersects the given points on the function.

3) $y = -x^2 + x + 2;\ (-2, -4), (1, 2)$



4) $y = -\dfrac{1}{x};\ (1, -1), \left(3, -\dfrac{1}{3}\right)$



For each problem, find the average rate of change of the function over the given interval.

5) $y = x^2 + 2;\ \ [-2, -\frac{3}{2}]$

6) $y = 2x^2 - 2x + 1;\ \ [-1, -\frac{1}{2}]$

7) $y = -\frac{1}{x + 2};\ \ [-1, -\frac{1}{2}]$

8) $y = 2x^2 + x + 2;\ \ [0, \frac{1}{2}]$

For each problem, find the equation of the secant line that intersects the given points on the function.

9) $y = -x^2 - 2;\ (1, -3), \left(\frac{3}{2}, -\frac{17}{4}\right)$

10) $y = \frac{1}{x + 3};\ \left(-1, \frac{1}{2}\right), \left(-\frac{1}{2}, \frac{2}{5}\right)$

11) $y = \frac{1}{x - 1};\ \left(-2, -\frac{1}{3}\right), \left(-\frac{3}{2}, -\frac{2}{5}\right)$

12) $y = -\frac{1}{x};\ (1, -1), \left(\frac{3}{2}, -\frac{2}{3}\right)$

Critical thinking question:

13) The police have accused a driver of breaking the speed limit of 60 miles per hour. As proof, they provide two photographs. One photo shows the driver's car passing a toll booth at exactly 6 PM. The second photo shows the driver's car passing another toll both 31 miles down the highway at exactly 6:30 PM. Does the photo evidence prove that the driver broke the speed limit during this time?

- Infinite Calculus Name________________________________

Definition of the Derivative

Date________________ Period____

Use the definition of the derivative to find the derivative of each function with respect to *x*.

1) $y = -2x + 5$

2) $f(x) = -4x - 2$

3) $y = 4x^2 + 1$

4) $f(x) = -3x^2 + 4$

5) $y = -4x^2 - 5x - 2$

6) $y = 3x^2 + 3x + 3$

7) $y = \sqrt{-3x - 5}$

8) $f(x) = \sqrt{4x - 5}$

9) $y = \dfrac{1}{x + 2}$

10) $f(x) = -\dfrac{2}{2x - 1}$

Critical thinking question:

11) Use the definition of the derivative to show that $f'(0)$ does not exist where $f(x) = |x|$.

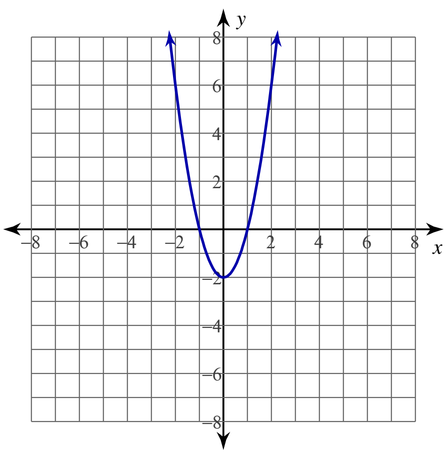
- Infinite Calculus Name________________________________

Instantaneous Rates of Change

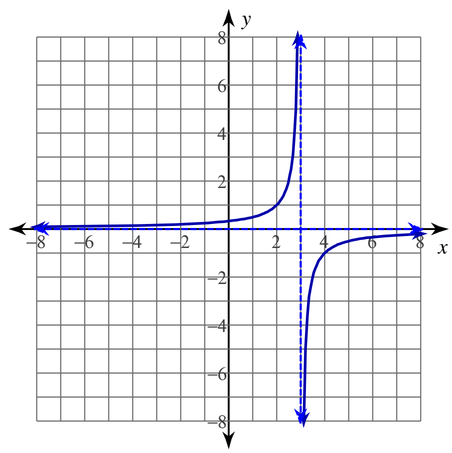
Date________________ Period____

For each problem, find the average rate of change of the function over the given interval and also find the instantaneous rate of change at the leftmost value of the given interval.

1) $y = 2x^2 - 2;\quad [1, \frac{3}{2}]$

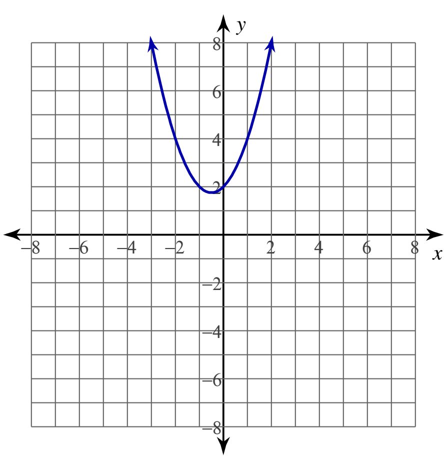


2) $y = -\frac{1}{x - 3};\quad [0, \frac{1}{2}]$

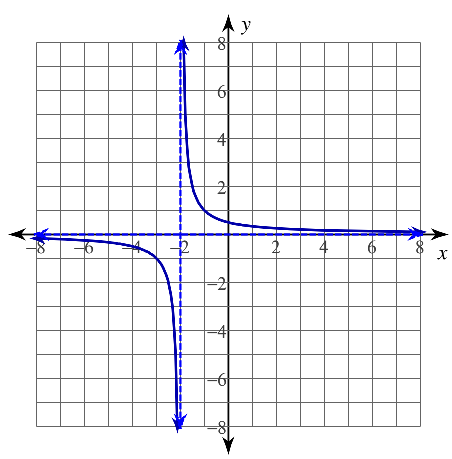


For each problem, find the equation of the secant line that intersects the given points on the function and also find the equation of the tangent line to the function at the leftmost given point. Sketch both lines for comparison.

3) $y = x^2 + x + 2;\ (-1, 2), \left(-\frac{1}{2}, \frac{7}{4}\right)$



4) $y = \frac{1}{x + 2};\ (-1, 1), \left(-\frac{1}{2}, \frac{2}{3}\right)$



- Infinite Calculus Name________________________________

Differentiation - Power, Constant, and Sum Rules

Date________________ Period____

Differentiate each function with respect to *x*.

1) $y = 5$

2) $f(x) = 5x^{18}$

3) $y = 4x^5 + x$

4) $f(x) = 4x^4 - 5x - 3$

5) $y = 3x^{\frac{5}{4}}$

6) $y = \frac{5}{4}x^{\frac{2}{3}}$

7) $y = -4x^{-5}$

8) $y = \frac{3}{x^3}$

9) $y = x^{\frac{2}{3}}$

10) $f(x) = -2\sqrt[4]{x}$

11) $y = \frac{2}{3}x^4 + 5x - x^{-3}$

12) $y = -\frac{1}{2}x^4 + 3x^{\frac{5}{3}} + 2x$

Differentiate each function with respect to the given variable.

13) $y = -3r^5 - 5r^2$

14) $f(s) = -\frac{3}{s^2} - \frac{4}{s^4}$

15) $f(x) = \frac{2}{3}x^{\frac{3}{2}} - \frac{3}{4}x^{\frac{3}{5}}$

16) $h(s) = \sqrt{2} \cdot \sqrt[3]{s} + \sqrt{2} \cdot \sqrt[5]{s}$

Differentiate each function with respect to *x*. Problems may contain constants a, b, and c.

17) $y = 5c$

18) $y = 4ax^{3a} - bx^{3c}$

- Infinite Calculus

Name________________________________

Higher Order Derivatives

Date________________ Period____

For each problem, find the indicated derivative with respect to *x*.

1) $y = -x^2$ Find $\dfrac{d^2y}{dx^2}$

2) $f(x) = 4x^3$ Find f''

3) $y = -4x$ Find $\dfrac{d^3y}{dx^3}$

4) $f(x) = 5x^4$ Find f'''

5) $y = -5x^4$ Find $\dfrac{d^4y}{dx^4}$

6) $y = 3x^5 - 2x$ Find $\dfrac{d^3y}{dx^3}$

7) $y = -2x^3 - 4x^{-3}$ Find $\dfrac{d^3y}{dx^3}$

8) $y = -x^2 + 2\sqrt[5]{x^2}$ Find $\dfrac{d^3y}{dx^3}$

Critical thinking questions. Find the indicated derivatives with respect to *x*.

9) $y = 99x^{99}$ Find $\dfrac{d^{100}y}{dx^{100}}$

10) $f(x) = x^{99}$ Find $f^{(99)}$

- Infinite Calculus Differentiation -

Name________________________________

Date________________ Period____

Product Rule **Differentiate each function with respect to *x*.**

1) $y = -x^3(3x^4 - 2)$

2) $f(x) = x^2(-3x^2 - 2)$

3) $y = (-2x^4 - 3)(-2x^2 + 1)$

4) $f(x) = (2x^4 - 3)(x^2 + 1)$

5) $f(x) = (5x^5 + 5)(-2x^5 - 3)$

6) $f(x) = (-3 + x^{-3})(-4x^3 + 3)$

7) $y = (-2x^4 + 5x^2 + 4)(-3x^2 + 2)$

8) $y = (x^4 + 3)(-4x^5 + 5x^4 + 5)$

9) $y = (5x^4 - 3x^2 - 1)(-5x^2 + 3)$

10) $f(x) = \left(-10x^2 - 7\sqrt[5]{x^2} + 9\right)(2x^3 + 4)$

11) $y = (5 + 3x^{-2})(4x^5 + 6x^3 + 10)$

12) $y = (-6x^4 + 2 + 6x^{-4})(6x^4 + 7)$

13) $f(x) = \left(-7x^4 + 10x^{\frac{2}{5}} + 8\right)(x^2 + 10)$

Critical thinking question:

14) A classmate claims that $(f \cdot g)' = f' \cdot g'$ for any functions f and g. Show an example that proves your classmate wrong.

- Infinite Calculus

Name________________________________

Differentiation - Quotient Rule

Date________________ Period____

Differentiate each function with respect to *x*.

1) $y = \frac{2}{2x^4 - 5}$

2) $f(x) = \frac{2}{x^5 - 5}$

3) $f(x) = \frac{5}{4x^3 + 4}$

4) $y = \frac{4x^3 - 3x^2}{4x^5 - 4}$

5) $y = \frac{3x^4 + 2}{3x^3 - 2}$

6) $y = \frac{4x^5 + 2x^2}{3x^4 + 5}$

7) $y = \frac{4x^5 + x^2 + 4}{5x^2 - 2}$

8) $y = \frac{3x^4 + 5x^3 - 5}{2x^4 - 4}$

9) $y = \dfrac{x^3 - x^2 - 3}{x^5 + 3}$

10) $y = \dfrac{x^4 + 6}{3 - 4x^{-4}}$

11) $y = \dfrac{4x^4 - 4x^2 + 5}{2x^{\frac{5}{3}} + 3}$

Critical thinking question:

12) A classmate claims that $\left(\dfrac{f}{g}\right)' = \dfrac{f'}{g'}$ for any functions f and g. Show an example that proves your classmate wrong.

- Infinite Calculus Differentiation -

Name________________________________

Date________________ Period____

Chain Rule **Differentiate each function with respect to *x*.**

1) $y = (x^3 + 3)^5$

2) $y = (-3x^5 + 1)^3$

3) $y = (-5x^3 - 3)^3$

4) $y = (5x^2 + 3)^4$

5) $f(x) = \sqrt[4]{-3x^4 - 2}$

6) $f(x) = \sqrt{-2x^2 + 1}$

7) $f(x) = \sqrt[3]{-2x^4 + 5}$

8) $y = (-x^4 - 3)^{-2}$

9) $y = (3x^3 + 1)(-4x^2 - 3)^4$

10) $y = \dfrac{(x^3 + 4)^5}{3x^4 - 2}$

11) $y = ((x + 5)^5 - 1)^4$

12) $y = (5x^3 - 3)^5 \sqrt[4]{-4x^5 - 3}$

Critical thinking question:

13) Give a function that requires three applications of the chain rule to differentiate. Then differentiate the function.

Name________________________________

Differentiation Rules, with Tables

Date________________ Period____

For each problem, you are given a table containing some values of differentiable functions $f(x)$, $g(x)$ and their derivatives. Use the table data and the rules of differentiation to solve each problem.

1)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	1	1	2
2	3	0	3	0
3	2	−1	1	−2

Given $h(x) = f(x) + g(x)$, find $h'(1)$

2)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	−1	1	1
2	1	$\frac{1}{2}$	2	1
3	3	2	3	1

Given $h(x) = f(x) - g(x)$, find $h'(2)$

3)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	−1	1	2
2	1	$\frac{1}{2}$	3	0
3	3	2	1	−2

Given $h(x) = f(x) \cdot g(x)$, find $h'(3)$

4)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	−1	2	−1
2	2	−1	1	0
3	1	−1	2	1

Given $h(x) = \frac{f(x)}{g(x)}$, find $h'(3)$

5)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	5	−1	1	2
2	4	−1	3	$\frac{3}{2}$
3	3	−1	4	1
4	2	−1	5	1
5	1	0	6	$-\frac{1}{2}$
6	2	1	4	−2

Part 1) Given $h_1(x) = f(x) + g(x)$, find $h_1'(2)$

Part 2) Given $h_2(x) = f(x) - g(x)$, find $h_2'(3)$

Part 3) Given $h_3(x) = f(x) \cdot g(x)$, find $h_3'(4)$

Part 4) Given $h_4(x) = \dfrac{f(x)}{g(x)}$, find $h_4'(2)$

Part 5) Given $h_5(x) = (f(x))^2$, find $h_5'(2)$

Part 6) Given $h_6(x) = f(g(x))$, find $h_6'(6)$

Name_______________________________

Differentiation - Trigonometric Functions

Date________________ Period____

Differentiate each function with respect to *x*.

1) $f(x) = \sin 2x^3$

2) $y = \tan 5x^3$

3) $y = \sec 4x^5$

4) $y = \csc 5x^5$

5) $y = (2x^5 + 3)\cos x^2$

6) $y = \dfrac{-2x^2 - 5}{\cos 2x^3}$

7) $f(x) = \sin^3 x^5$

8) $f(x) = \cos (-3x^2 + 2)^2$

- Infinite Calculus Name________________________________

Differentiation - Inverse Trigonometric Functions

Date________________ Period____

Differentiate each function with respect to *x*.

1) $y = \cos^{-1} -5x^3$

2) $y = \sin^{-1} -2x^2$

3) $y = \tan^{-1} 2x^4$

4) $y = \csc^{-1} 4x^2$

5) $y = (\sin^{-1} 5x^2)^3$

6) $y = \sin^{-1} (3x^5 + 1)^3$

7) $y = (\cos^{-1} 4x^2)^2$

8) $y = \cos^{-1} (-2x^3 - 3)^3$

- Infinite Calculus Name________________________

Differentiation - Natural Logs and Exponentials

Date________________ Period____

Differentiate each function with respect to *x*.

1) $y = \ln x^3$

2) $y = e^{2x^3}$

3) $y = \ln \ln 2x^4$

4) $y = \ln \ln 3x^3$

5) $y = \cos \ln 4x^3$

6) $y = e^{e^{3x^2}}$

7) $y = e^{(4x^3 + 5)^2}$

8) $y = \ln 4x^2 \cdot (-x^3 - 4)$

9) $y = \ln \left(-\frac{4x^4}{x^3 - 3}\right)^5$

10) $y = \frac{e^{5x^4}}{e^{4x^2 + 3}}$

- Infinite Calculus Differentiation - Logs and Exponentials **Differentiate each function with respect to *x*.**

Name___________________________________

Date__________________ Period____

1) $y = 4^{4x^4}$

2) $y = 4^{-5x^3}$

3) $y = \log_3 3x^2$

4) $y = \log_2 4x^2$

5) $y = \log_3 (3x^5 + 5)^5$

6) $y = \log_5 (-5x^3 - 2)^3$

7) $y = (4^{x^3} + 2)^3$

8) $y = 3^{(x^4 + 1)^3}$

9) $y = 3^{\cos 3x^4}$

10) $y = \log_5 \tan 4x^4$

9) $y = \dfrac{\sqrt{2x^3 + 3}}{(x^4 - 3)^3}$

10) $y = (2x^2 - 5)^3 \sqrt{x^2 - 2}$

Use logarithmic differentiation to differentiate each function with respect to *x*. You do not need to simplify or substitute for *y*.

11) $y = \dfrac{(5x - 4)^4}{(3x^2 + 5)^5 \cdot (5x^4 - 3)^3}$

12) $y = (x + 2)^4 \cdot (2x - 5)^2 \cdot (5x + 1)^3$

13) $y = (5x^5 + 2)^2 \cdot (3x^3 - 1)^3 \cdot (3x - 1)^4$

14) $y = \dfrac{(x^2 + 3)^4}{(5x^5 - 2)^5 \cdot (3x^2 - 5)^2}$

15) $y = (3x^3 - 4)^5 \cdot (3x - 1)^3 \cdot (5x^3 - 2)^2 \cdot (x + 3)^4$

16) $y = \dfrac{(4x^2 - 5)^2}{(2x - 3)^4 \cdot (5x^4 - 2)^5 \cdot (3x^2 - 4)^3}$

- Infinite Calculus Name_______________________________

Logarithmic Differentiation

Date________________ Period____

Use logarithmic differentiation to differentiate each function with respect to *x*.

1) $y = 2x^{2x}$

2) $y = 5x^{5x}$

3) $y = 3x^{3x}$

4) $y = 4x^{x^4}$

5) $y = (3x^4 + 4)^3\sqrt{5x^3 + 1}$

6) $y = (x^5 + 5)^2\sqrt{2x^2 + 3}$

7) $y = \dfrac{(3x^4 - 2)^5}{(3x^3 + 4)^2}$

8) $y = \sqrt{3x^2 + 1}\,(3x^4 + 1)^3$

9) $y = \dfrac{\sqrt{2x^3 + 3}}{(x^4 - 3)^3}$

10) $y = (2x^2 - 5)^3 \sqrt{x^2 - 2}$

Use logarithmic differentiation to differentiate each function with respect to *x*. You do not need to simplify or substitute for *y*.

11) $y = \dfrac{(5x - 4)^4}{(3x^2 + 5)^5 \cdot (5x^4 - 3)^3}$

12) $y = (x + 2)^4 \cdot (2x - 5)^2 \cdot (5x + 1)^3$

13) $y = (5x^5 + 2)^2 \cdot (3x^3 - 1)^3 \cdot (3x - 1)^4$

14) $y = \dfrac{(x^2 + 3)^4}{(5x^5 - 2)^5 \cdot (3x^2 - 5)^2}$

15) $y = (3x^3 - 4)^5 \cdot (3x - 1)^3 \cdot (5x^3 - 2)^2 \cdot (x + 3)^4$

16) $y = \dfrac{(4x^2 - 5)^2}{(2x - 3)^4 \cdot (5x^4 - 2)^5 \cdot (3x^2 - 4)^3}$

- Infinite Calculus

Name________________________________

Implicit Differentiation

Date________________ Period____

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y.

1) $2x^3 = 2y^2 + 5$

2) $3x^2 + 3y^2 = 2$

3) $5y^2 = 2x^3 - 5y$

4) $4x^2 = 2y^3 + 4y$

5) $5x^3 = -3xy + 2$

6) $1 = 3x + 2x^2y^2$

7) $3x^2y^2 = 4x^2 - 4xy$

8) $5x^3 + xy^2 = 5x^3y^3$

9) $2x^3 = (3xy + 1)^2$

10) $x^2 = (4x^2y^3 + 1)^2$

11) $\sin 2x^2y^3 = 3x^3 + 1$

12) $3x^2 + 3 = \ln 5xy^2$

For each problem, use implicit differentiation to find $\frac{d^2y}{dx^2}$ in terms of x and y.

13) $4y^2 + 2 = 3x^2$

14) $5 = 4x^2 + 5y^2$

Critical thinking question:

15) Use three strategies to find $\frac{dy}{dx}$ in terms of x and y, where $\frac{3x^2}{4y} = x$. Strategy 1: Use implicit differentiation directly on the given equation. Strategy 2: Multiply both sides of the given equation by the denominator of the left side, then use implicit differentiation. Strategy 3: Solve for y, then differentiate. Do your three answers look the same? If not, how can you show that they are all correct answers?

- Infinite Calculus Name________________________

Derivatives of Inverse Functions

Date________________ Period____

For each problem, find $(f^{-1})'(x)$ by direct computation.

1) $f(x) = -3x + 3$

2) $f(x) = -2x + 3$

For each problem, find $(f^{-1})'(x)$ by using the theorem $(f^{-1})'(x) = \dfrac{1}{f'(f^{-1}(x))}$

3) $f(x) = -5x + 1$

4) $f(x) = -2x + 2$

5) $f(x) = \sqrt{-2x - 3}$

6) $f(x) = -4x^3 - 4$

For each problem, find $(f^{-1})'(x)$ by using the formula $\dfrac{dy}{dx} = \dfrac{1}{\frac{dx}{dy}}$, where $y = f^{-1}(x)$

7) $f(x) = x^7 + x - 3$

8) $f(x) = 3x^5 + 2x + 5$