- Infinite Calculus Evaluating   Name____________________

Limits **Evaluate each limit.**

Date________________ Period____

1) $\lim_{x \to -1} 5$

5

2) $\lim_{x \to -\frac{5}{2}} (-x + 2)$

$\frac{9}{2}$

3) $\lim_{x \to 2} (x^3 - x^2 - 4)$

0

4) $\lim_{x \to 1} \left(-\frac{x^2}{2} + 2x + 4\right)$

$\frac{11}{2}$

5) $\lim_{x \to 3} -\sqrt{x + 3}$

$-\sqrt{6}$

6) $\lim_{x \to \frac{3}{2}} -\sqrt{2x + 4}$

$-\sqrt{7}$

7) $\lim_{x \to 1} -\frac{x - 4}{x^2 - 6x + 8}$

1

8) $\lim_{x \to \frac{3}{2}} \frac{-x - 3}{x^2 + x + 1}$

$-\frac{18}{19}$

9) $\lim_{x \to \pi} \sin(x)$

0

10) $\lim_{x \to \frac{3\pi}{4}} 2\cos(x)$

$-\sqrt{2}$

Critical thinking questions:

11) Give an example of a limit that evaluates to 4.

Many answers. Ex: $\lim_{x \to 4} x$

12) Give an example of a limit of a quadratic function where the limit evaluates to 9.

Many answers. Ex: $\lim_{x \to 3} x^2$

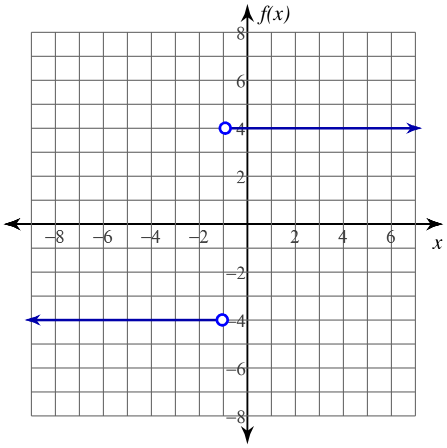
- Infinite Calculus Evaluating Limits

Name________________________________

Date________________ Period____

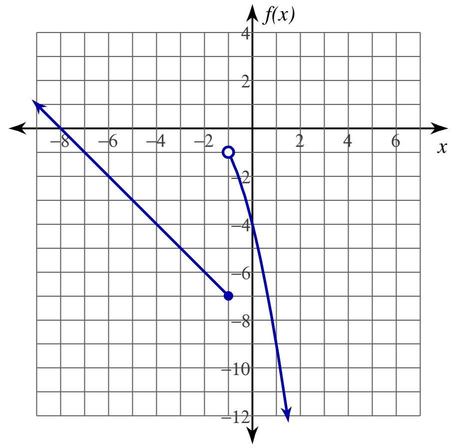
Evaluate each limit.

1) $\lim_{x \to -1^+} \frac{4x + 4}{|x + 1|}$



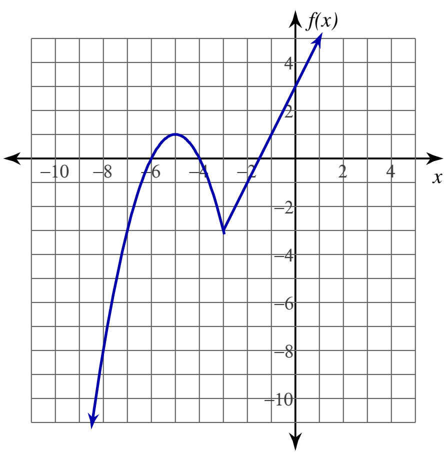
4

2) $\lim_{x \to -1^-} f(x),\ f(x) = \begin{cases} -x - 8, & x \le -1 \\ -x^2 - 4x - 4, & x > -1 \end{cases}$



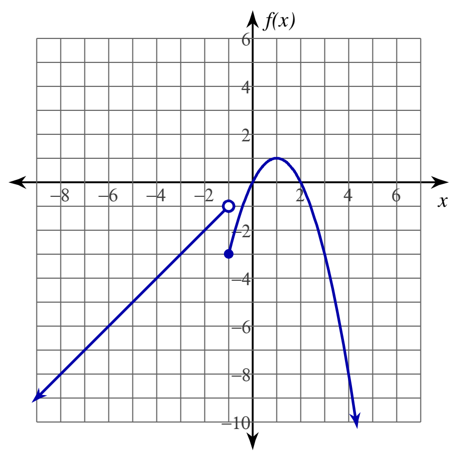
-7

3) $\lim_{x \to -3} f(x),\ f(x) = \begin{cases} -x^2 - 10x - 24, & x \le -3 \\ 2x + 3, & x > -3 \end{cases}$



-3

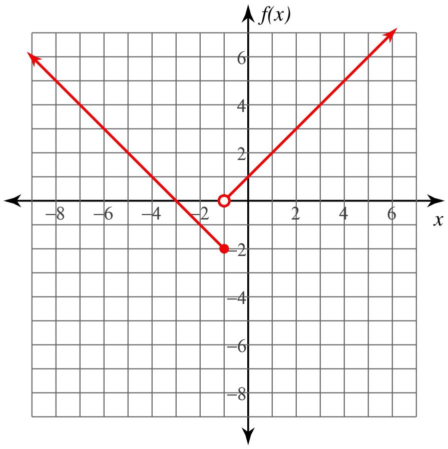
4) $\lim_{x \to -1} f(x),\ f(x) = \begin{cases} x, & x < -1 \\ -x^2 + 2x, & x \ge -1 \end{cases}$



Does not exist.

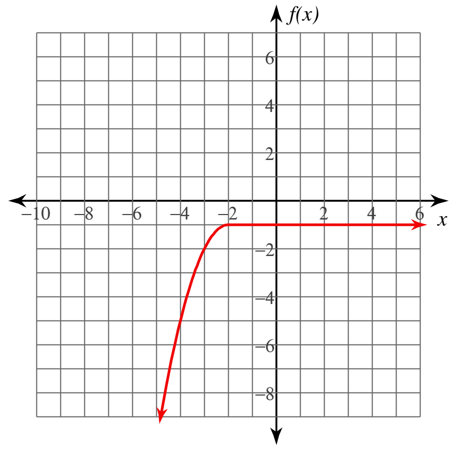
Evaluate each limit. You may use the provided graph to sketch the function.

5) $\lim\limits_{x \to -1^-} f(x),\ f(x) = \begin{cases} -x - 3, & x \le -1 \\ x + 1, & x > -1 \end{cases}$



−2

6) $\lim\limits_{x \to -2} f(x),\ f(x) = \begin{cases} -x^2 - 4x - 5, & x \le -2 \\ -1, & x > -2 \end{cases}$



−1

Evaluate each limit.

7) $\lim\limits_{x \to 0^+} f(x),\ f(x) = \begin{cases} 1, & x \le 0 \\ -x^2 + 4x - 3, & x > 0 \end{cases}$

−3

8) $\lim\limits_{x \to 0^-} \dfrac{|x|}{x}$

−1

9) $\lim\limits_{x \to 0^+} \lfloor -2x + 1 \rfloor$

0

10) $\lim\limits_{x \to 1} f(x),\ f(x) = \begin{cases} \dfrac{x}{2} + \dfrac{9}{2}, & x < 1 \\ x^2 - 6x + 10, & x \ge 1 \end{cases}$

5

11) $\lim\limits_{x \to -1} \dfrac{3|x + 1|}{x + 1}$

Does not exist.

12) $\lim\limits_{x \to -2} f(x),\ f(x) = \begin{cases} x^2, & x \le -2 \\ -\dfrac{x}{2} + 3, & x > -2 \end{cases}$

4

Critical thinking questions:

13) Give an example of a two-sided limit of a piecewise function where the limit does not exist.

Many answers. Ex: $\lim\limits_{x \to 1} f(x),\ f(x) = \begin{cases} 0, & x < 1 \\ x, & x \ge 1 \end{cases}$

14) Given an example of a two-sided limit of a function with an absolute value where the limit does not exist.

Many answers. Ex: $\lim\limits_{x \to 0} \dfrac{|x|}{x}$

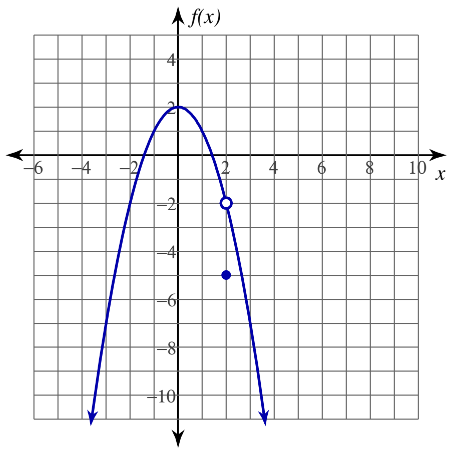
Infinite Calculus Evaluating Limits

Name________________________________

Date________________ Period____

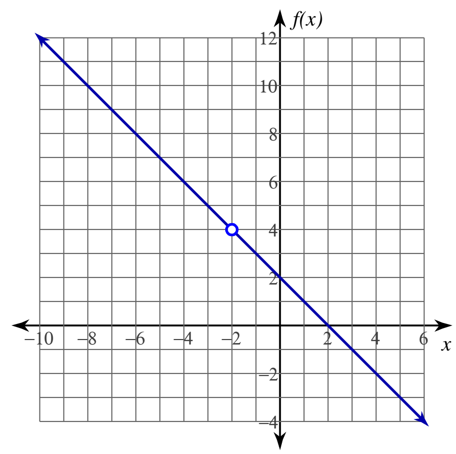
Evaluate each limit.

1) $\lim_{x \to 2} f(x),\ f(x) = \begin{cases} -x^2 + 2, & x \neq 2 \\ -5, & x = 2 \end{cases}$



-2

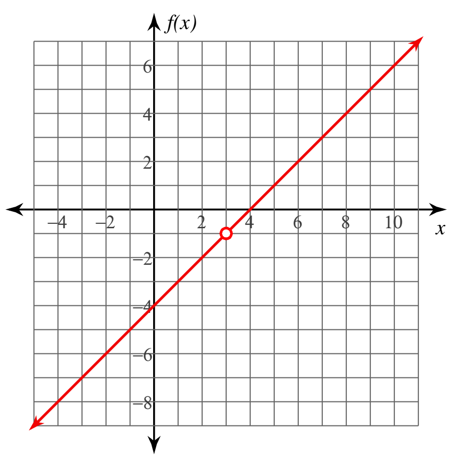
2) $\lim_{x \to -2} -\dfrac{x^2 - 4}{x + 2}$



4

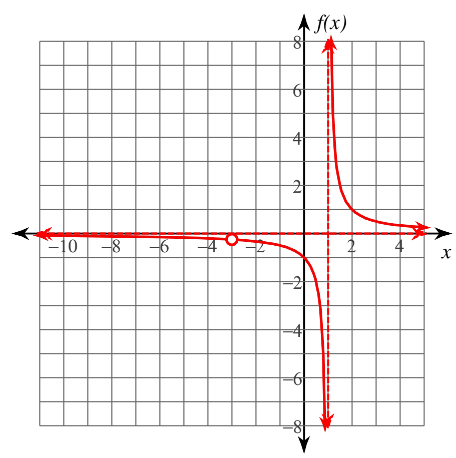
Evaluate each limit. You may use the provided graph to sketch the function.

3) $\lim_{x \to 3} \dfrac{x^2 - 7x + 12}{x - 3}$



-1

4) $\lim_{x \to -3} \dfrac{x + 3}{x^2 + 2x - 3}$



$-\dfrac{1}{4}$

Evaluate each limit.

5) $\lim_{x \to 0} f(x),\ f(x) = \begin{cases} x + 1, & x \neq 0 \\ 2, & x = 0 \end{cases}$

1

6) $\lim_{x \to 3} f(x),\ f(x) = \begin{cases} 2 + \dfrac{x}{2}, & x \neq 3 \\ 2, & x = 3 \end{cases}$

$\dfrac{7}{2}$

7) $\lim_{x \to 1} -\frac{x^2 - 1}{x - 1}$

-2

8) $\lim_{x \to 5} -\frac{x^2 - 5x}{x - 5}$

-5

9) $\lim_{x \to 2} -\frac{x^2 - x - 2}{x - 2}$

-3

10) $\lim_{x \to -5} \frac{x^2 + 3x - 10}{x + 5}$

-7

11) $\lim_{x \to 0} \frac{\frac{1}{-4 + x} + \frac{1}{4}}{x}$

$-\frac{1}{16}$

12) $\lim_{x \to -3} \frac{x}{\frac{1}{3 + x} - \frac{1}{3}}$

0

13) $\lim_{x \to 5} \frac{x - 5}{\sqrt{x + 4} - 3}$

6

14) $\lim_{x \to 3} \frac{\sqrt{x + 6} - 3}{x - 3}$

$\frac{1}{6}$

Critical thinking questions:

15) Give an example of a limit of a rational function where the limit at -1 exists, but the rational function is undefined at -1.

Many answers. Ex: $\lim_{x \to -1} \frac{x^2 - 1}{x + 1}$

16) Give two values of a where the limit cannot be solved using direct evaluation. Give one value of a where the limit can be solved using direct evaluation.

$\lim_{x \to a} \frac{x}{\frac{1}{-2 + x} + \frac{1}{2}}$

No direct eval: a=0,2 Direct eval: a=any other number

- Infinite Calculus Evaluating Limits

Name________________________________

Date________________ Period____

Evaluate each limit.

1) $\lim_{x \to 0} \frac{1 - \sin\left(\frac{\pi}{2} - x\right)}{x}$

0

2) $\lim_{x \to 0} \frac{\cos\left(\frac{\pi}{2} - x\right)}{x}$

1

3) $\lim_{x \to 0} \frac{\tan(x)}{3x}$

$\frac{1}{3}$

4) $\lim_{x \to 0} \frac{\sin(x)}{\sin(4x)}$

$\frac{1}{4}$

5) $\lim_{x \to 0} \frac{1 - \cos(2x)}{4x}$

0

6) $\lim_{x \to 0} \frac{\sin^2(2x)}{x^2}$

4

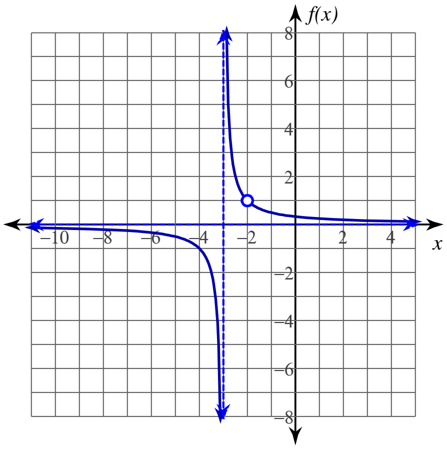
Infinite Calculus Evaluating Limits

Name________________________________

Date________________ Period____

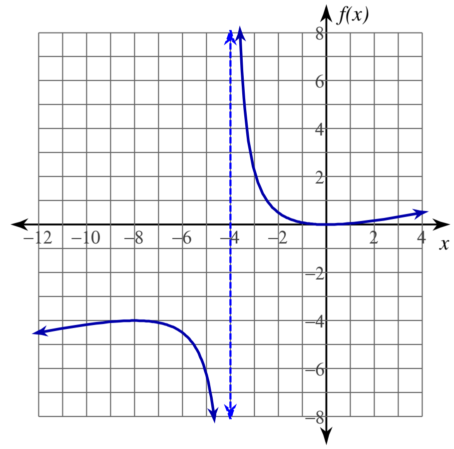
Evaluate each limit.

1) $\lim_{x \to -3^+} \frac{x+2}{x^2+5x+6}$



∞

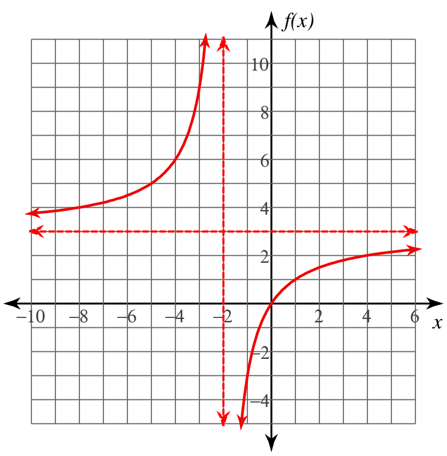
2) $\lim_{x \to -4} \frac{x^2}{4x+16}$



Does not exist.

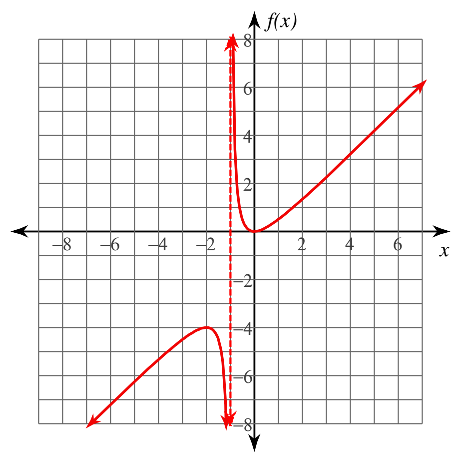
Evaluate each limit. You may use the provided graph to sketch the function.

3) $\lim_{x \to -2^+} \frac{3x}{x+2}$



$-\infty$

4) $\lim_{x \to -1^+} \frac{x^2}{x+1}$



∞

Evaluate each limit.

5) $\lim_{x \to -3^-} \frac{2x}{x+3}$

∞

6) $\lim_{x \to -2^+} \frac{1}{x^2 - 4}$

$-\infty$

7) $\lim_{x \to 3^-} -\frac{4x}{x-3}$

∞

8) $\lim_{x \to 1} -\frac{3}{x-1}$

Does not exist.

9) $\lim_{x \to -2^-} \frac{x+2}{x^2 + x - 2}$

$-\frac{1}{3}$ (distractor case, limit exists)

10) $\lim_{x \to -3^-} -\frac{2}{x+3}$

∞

11) $\lim_{x \to \frac{\pi}{4}^-} 2\sec(2x)$

∞

12) $\lim_{x \to \frac{3\pi}{4}^+} 2\tan(2x)$

$-\infty$

Critical thinking questions:

13) Give an example of a right-sided limit that goes to ∞ as x goes to 5.

Many answers. Ex: $\lim_{x \to 5^+} \frac{1}{x-5}$

14) Give an example of a left-sided limit that goes to ∞ as x goes to 5.

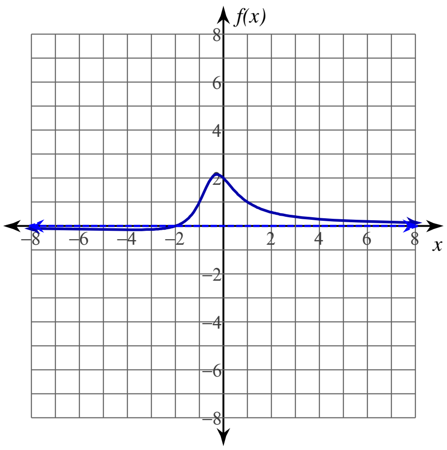
Many answers. Ex: $\lim_{x \to 5^-} -\frac{1}{x-5}$

Infinite Calculus Evaluating Limits **Evaluate each limit.**

Name________________________________

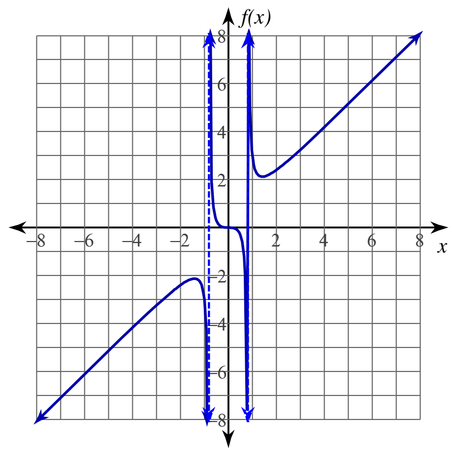
Date________________ Period____

1) $\lim_{x \to -\infty} \frac{x + 2}{x^2 + x + 1}$



0

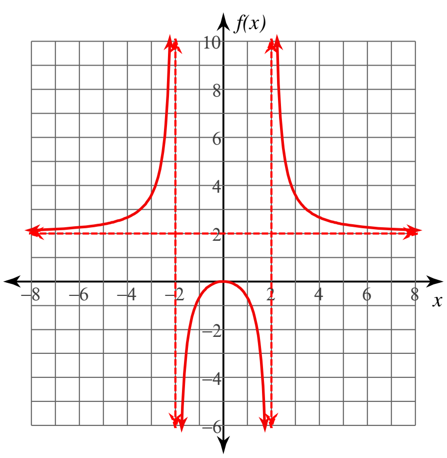
2) $\lim_{x \to -\infty} \frac{3x^3}{3x^2 - 2}$



$-\infty$

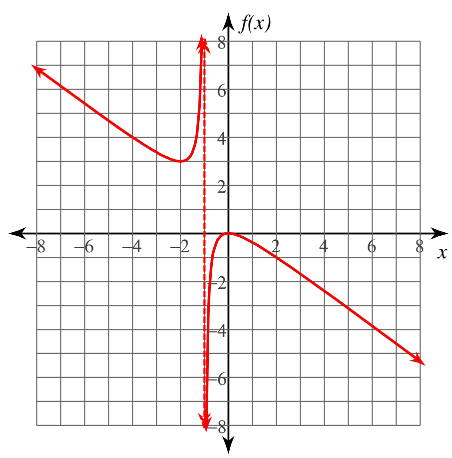
Evaluate each limit. You may use the provided graph to sketch the function.

3) $\lim_{x \to -\infty} \frac{2x^2}{x^2 - 4}$



2

4) $\lim_{x \to \infty} -\frac{3x^2}{4x + 4}$



$-\infty$

Evaluate each limit.

5) $\lim_{x \to -\infty} (x^3 - 4x^2 + 5)$

$-\infty$

6) $\lim_{x \to \infty} \frac{2x^3}{3x^2 - 4}$

∞

7) $\lim_{x \to \infty} \frac{x^3}{4x^2 + 3}$

∞

8) $\lim_{x \to \infty} \frac{x + 1}{2x^2 + 2x + 1}$

0

9) $\lim_{x \to -\infty} \frac{\sqrt{2x^2 + 3}}{2x + 3}$

$-\frac{\sqrt{2}}{2}$

10) $\lim_{x \to -\infty} \frac{\sqrt{2x^2 + 1}}{4x + 2}$

$-\frac{\sqrt{2}}{4}$

11) $\lim_{x \to \infty} \left(-\frac{\ln x}{x^4} + 1\right)$

1

12) $\lim_{x \to \infty} (-e^{-3x} - 1)$

-1

13) $\lim_{x \to \infty} (e^x - 3)$

∞

14) $\lim_{x \to -\infty} -e^{-4x}$

$-\infty$

15) $\lim_{x \to \infty} \cos(2x)$

Does not exist. Oscillates.

16) $\lim_{x \to -\infty} \frac{x}{\cos(-3x)}$

Does not exist. Oscillates.

17) $\lim_{x \to \infty} -\frac{2x}{\cos \frac{1}{x}}$

$-\infty$

18) $\lim_{x \to \infty} x\cos \frac{1}{x}$

∞

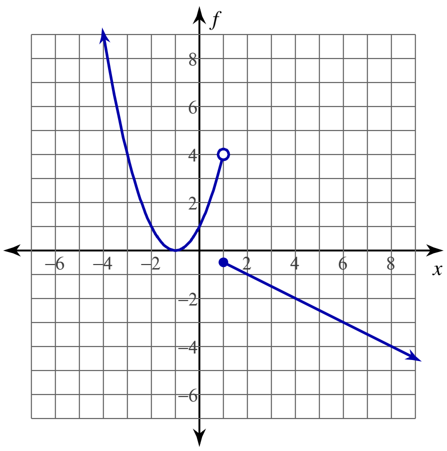
Name________________________________

Continuity

Date________________ Period____

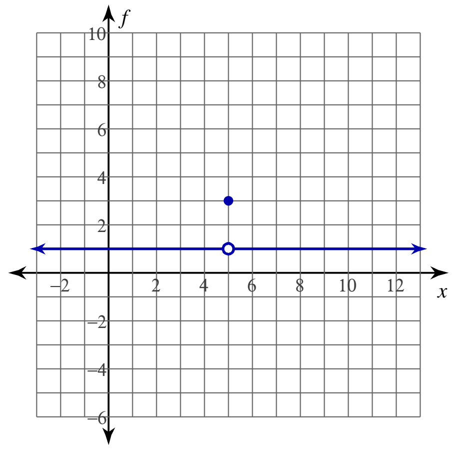
Find the intervals on which each function is continuous.

1) $f(x) = \begin{cases} x^2 + 2x + 1, & x < 1 \\ -\dfrac{x}{2}, & x \geq 1 \end{cases}$



$(-\infty, 1), [1, \infty)$

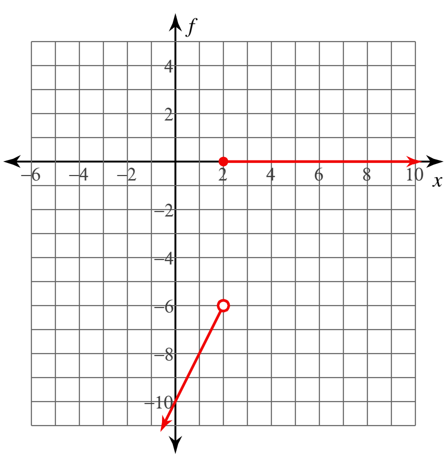
2) $f(x) = \begin{cases} 1, & x \neq 5 \\ 3, & x = 5 \end{cases}$



$(-\infty, 5), (5, \infty)$

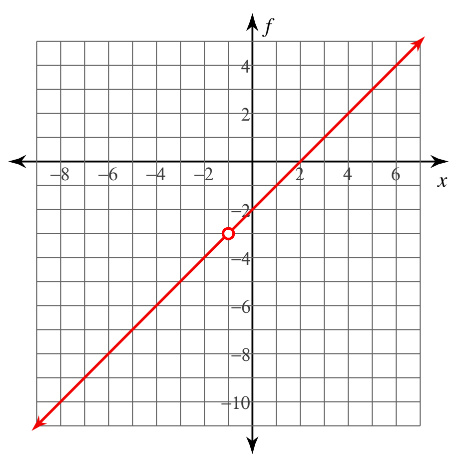
Find the intervals on which each function is continuous. You may use the provided graph to sketch the function.

3) $f(x) = \begin{cases} 2x - 10, & x < 2 \\ 0, & x \geq 2 \end{cases}$



$(-\infty, 2), [2, \infty)$

4) $f(x) = \dfrac{x^2 - x - 2}{x + 1}$



$(-\infty, -1), (-1, \infty)$

Find the intervals on which each function is continuous.

5) $f(x) = \frac{x^2}{2x + 4}$

$(-\infty, -2), (-2, \infty)$

6) $f(x) = \begin{cases} -\frac{x}{2} - \frac{7}{2}, & x \le 0 \\ -x^2 + 2x - 2, & x > 0 \end{cases}$

$(-\infty, 0], (0, \infty)$

7) $f(x) = -\frac{x^2 - x - 12}{x + 3}$

$(-\infty, -3), (-3, \infty)$

8) $f(x) = \frac{x^2 - x - 6}{x + 2}$

$(-\infty, -2), (-2, \infty)$

Determine if each function is continuous. If the function is not continuous, find the *x*-axis location of and classify each discontinuity.

9) $f(x) = -\frac{x^2}{2x + 4}$

Essential discontinuity at: $x = -2$

10) $f(x) = \frac{x + 1}{x^2 - x - 2}$

Removable discontinuity at: $x = -1$
Essential discontinuity at: $x = 2$

11) $f(x) = \frac{x + 1}{x^2 + x + 1}$

Continuous

12) $f(x) = -\frac{x^2}{x - 1}$

Essential discontinuity at: $x = 1$

13) $f(x) = \begin{cases} x^2 - 4x + 3, & x \ne 0 \\ 3, & x = 0 \end{cases}$

Continuous

14) $f(x) = \begin{cases} -x^2, & x \ne 1 \\ 0, & x = 1 \end{cases}$

Removable discontinuity at: $x = 1$

Critical thinking questions:

15) Give an example of a function with discontinuities at $x = 1$, 2, and 3.

Many answers. $\frac{1}{(x - 1)(x - 2)(x - 3)}$

16) Of the six basic trigonometric functions, which are continuous over all real numbers? Which are not? What types of discontinuities are there?

Cont: sin, cos. Not cont: sec, csc, tan, cot. Essential.