

Unit Equations:

A unit equation is exactly what it sounds like; an equation that gives a relationship between 2 or more units. For example, **1 foot = 12 inches** and **9.81 m = 1 s²** are unit equations because they have an equal sign and 2 different units. While most of the unit equations that we will deal with in this class have only 2 units, it is important to note that there can be MORE than 2 units. A single unit equation having 4 separate units, and one which we will be using this semester, is: **0.08206 atm•L = 1 mol•K**. Even though it has a bunch of different units, this is still going to act like any other unit equation. As stated above, these unit equations show the relationship between 2 more units and as such, they can be used to change one unit in the relationship to another one. The relationship **1 gallon = 4 quarts** can be used to convert back and forth between gallons and quarts. (NOTE: Some unit equations are always true, like 1 foot = 12 inches, which is true no matter what. Other unit equations are only true in certain situations, like 9.81 m = 1 s², which is only true on earth. We'll come back to this when we discuss sig figs of conversion factors.)

The metric system

Things get somewhat trickier when we start talking about unit equations based on the metric system, though the whole thing is laid out for us. The charts in both the book and that I handed out in lecture set up 10 or more different generic "unit equations". For example, the charts tell us that for kilo, **1 k = 1x10³**. One problem with this is that there is only one unit, but we will let that go for now. If I want to talk about kilometers, I need to make the left side **1 km**, but remember that what I do to one side of an equation I must do to the other which means the other side must become **1x10³ m**. This makes the unit equation **1 km = 1x10³ m**. The same would be true if we wanted to talk about kiloliters. We take the generic "unit equation" **k = 1x10³** and add liters to both giving **1 kL = 1x10³ L**. Want to talk about microseconds? Piece of cake! Take the generic "unit equation" for micro, **1 μ = 1x10⁻⁶**, add seconds to both sides and viola! you have the unit equation **1 μs = 1x10⁻⁶ s**. This is how you work the metric system, you memorize the 10 generic "unit equations" from the tables, and then just add the unit you want to both sides! If you do this, you cannot make a mistake!

Practice: Please set up unit equations for each of the following metric relationships

- a) Scoville (Sc) and kiloScoville (kSc)
- b) mole (mol) and millimole (mmol)
- c) megacalorie (Mcal) and calorie (cal)
- d) farad (f) and picofarad (pf)
- e) gigawatt (GW) and watt (W)
- f) nanotesla (nT) and tesla (T)

Conversion Factors:

Once you are comfortable with unit equations, conversion factors are next. In fact the unit equations are the hard part! Once you have a unit equation down, it is as simple as putting the left side under the right side for the first one, and then the right side under the left side for the other. No tricks! You will always get two conversion factors from any unit equation. Going back to my example of a

kilometer, the unit equation is **1 km = 1x10³ m**. Moving the left side under the right side, we get $\frac{1 \times 10^3 \text{ m}}{1 \text{ km}}$ as the first conversion

factor and moving the right side under the left side, we get $\frac{1 \text{ km}}{1 \times 10^3 \text{ m}}$ as the second conversion factor. We can then decide which

conversion factor to use in our dimensional analysis. Another example I gave was **0.08206 atm•L = 1 mol•K**. To get the conversion

factors from this, we go through the same process and we get $\frac{0.08206 \text{ atm} \cdot \text{L}}{1 \text{ mol} \cdot \text{K}}$ or $\frac{1 \text{ mol} \cdot \text{K}}{0.08206 \text{ atm} \cdot \text{L}}$.

Practice: Please give the 2 possible conversion factors for each of the following (the first 6 are from above)

- | | |
|---|--|
| a) Scoville (Sc) and kiloScoville (kSc) | f) nanotesla (nT) and tesla (T) |
| b) mole (mol) and millimole (mmol) | g) 4.184 J = 1 g•l °C (specific heat capacity of water) |
| c) megacalorie (Mcal) and calorie (cal) | h) 0.789 g = 1 mL (density of ethanol) |
| d) farad (f) and picofarad (pf) | i) 1 mol•l K = 8.314 J (one version of the gas constant) |
| e) gigawatt (GW) and watt (W) | j) 83 miles = 1 hour (the speed on my brother's ticket) |

Sig Figs and Conversion Factors:

We know how to determine how many sig figs are in a number, but we need to clear up some stuff about conversion factors. First, recall that in general, the number “1” does NOT count towards sig figs; it is considered a counted number. So now, the final rule on sig figs. If all units in the conversion factor are 1) in the same system of measure, and 2) measure the same quality, then the conversion factor has infinite sig figs. If not, then count the number of sig figs in the conversion as you normally would. For example, **3 ft = 1 yard**. Both **feet** and **yard** are in the standard system and both measure distance. Therefore, the conversion factor

$\frac{3 \text{ ft}}{1 \text{ yd}}$ infinite sig. figs. On your conversion sheet, you will see the unit equation **1 mL = 1 cm³**. Both **mL** and **cm³** are in the metric

system and both measure volume. Because both conditions are met, the conversion $\frac{1 \text{ mL}}{1 \text{ cm}^3}$ also has infinite sig figs. Let's now look

at two examples of when a conversion factor does NOT have infinite sig figs. For the metal iron, **7.87 g = 1 cm³** at room temperature. This relationship between mass and volume is different for different materials and is called the density. Both **grams** and **cm³** are in the metric system, but **grams** measures **mass** and **cm³** measures **volume**. Because the two units do NOT measure the same quality,

the conversion factor $\frac{7.87 \text{ g}}{1 \text{ cm}^3}$ does not have infinite sig figs, but rather has 3 sig figs. Another conversion is **1 stone = 6.350 kg**.

Stone is a **standard** measure of mass and **kg** is a **metric** measure of mass, therefore the conversion $\frac{1 \text{ stone}}{6.350 \text{ kg}}$ would NOT have infinite sig figs (not the same system of measure) but would have 4 sig figs when used in a calculation.

Practice: For each of the following, state the system of measure for each unit, the quality measured by each unit, and then give the number of sig figs in the conversion factor.

a) 1 J = 0.2390 cal

f) 1 kg = 2.20 lb

b) 1 km = 1.09x10³ yards

g) 65 mile = 1 hr

c) 1 L = 1x10⁻³ m³

h) 0.26420 L = 1 gal

d) 1 mL = 1.13 g

i) 196.96 g Au = 1 mole Au

e) 5280 ft = 1 mile

j) 1 ML = 1x10⁶ L

Using Conversion Factors:

Metric system:

Say I ask you to **convert 0.00671 seconds (s) to milliseconds (ms)**. The first thing you need to do is decide which is the base unit and which is the prefixed unit. Hopefully, you knew that seconds are the base unit (no prefix) and milliseconds are the prefixed unit (milli is the prefix). Next, you need to write out the general “unit equation” for the prefix and then add the base unit.

General “unit equation” → **1 m = 1x10⁻³**.

Actual unit equation → **1ms = 1x10⁻³ s**.

(A way to check that you have the correct unit equation is to verify that there is a 1 in front of the prefix (**ms**) and a power of 10 in front of the base unit (**s**.) Now that you have the unit equations, you can write the two possible conversion factors:

$$\frac{1 \text{ ms}}{1 \times 10^{-3} \text{ s}} \quad \text{or} \quad \frac{1 \times 10^{-3} \text{ s}}{1 \text{ ms}}$$

Finally, decide which of the conversion factors to use so that units cancel out. We are starting with 0.00671 seconds, so we want seconds on the bottom to cancel.

$$0.00671 \cancel{\text{ s}} \times \frac{1 \text{ ms}}{1 \times 10^{-3} \cancel{\text{ s}}} = 6.71 \text{ ms}$$

0.00671 has 3 sig figs. ms and s are both metric measures of time, so the conversion has infinite sig figs. Therefore, the answer as 3 sig figs.

Example: Convert 36.95 megawatts (MW) to watts (W)... (do we care what a watt is? NO!)

General “unit equation” $\rightarrow 1 \text{ M} = 1 \times 10^6$.

Actual unit equation $\rightarrow 1 \text{ MW} = 1 \times 10^6 \text{ W}$.

$$\frac{1 \text{ MW}}{1 \times 10^6 \text{ W}} \text{ or } \frac{1 \times 10^6 \text{ W}}{1 \text{ MW}} \rightarrow 36.95 \text{ MW} \times \frac{1 \times 10^6 \text{ W}}{1 \text{ MW}} = 3.695 \times 10^7 \text{ W}$$

Example: How many liters in $5.62 \times 10^5 \mu\text{L}$?

General “unit equation” $\rightarrow 1 \mu = 1 \times 10^{-6}$.

Actual unit equation $\rightarrow 1 \mu\text{L} = 1 \times 10^{-6} \text{ L}$.

$$\frac{1 \mu\text{L}}{1 \times 10^{-6} \text{ L}} \text{ or } \frac{1 \times 10^{-6} \text{ L}}{1 \mu\text{L}} \rightarrow 5.62 \times 10^5 \mu\text{L} \times \frac{1 \times 10^{-6} \text{ L}}{1 \mu\text{L}} = 0.562 \text{ L}$$

Practice: Perform the following conversions (write out the generic “unit equations”, unit equations, and both conversion factors for at least 6 of these):

- | | |
|---|---|
| a) 28.0 cm $\rightarrow$ m | i) $6.8 \times 10^4 \text{ ng} \rightarrow \text{g}$ |
| b) 1000. m $\rightarrow$ km | j) 8.54 g $\rightarrow$ cg |
| c) 9.28 m $\rightarrow$ mm | k) 25.0 mL $\rightarrow$ L |
| d) 10.68 g $\rightarrow$ mg | l) 22.4 L $\rightarrow$ μL |
| e) 4.5 m $\rightarrow$ dm | m) 0.65 Gs $\rightarrow$ s |
| f) 12 m $\rightarrow$ Mm | n) 5.5 kg $\rightarrow$ g |
| g) 23.6 kilojoules (kJ) to joules (J) | o) 0.468 TL $\rightarrow$ L |
| h) $1.6411 \times 10^7 \text{ pg} \rightarrow \text{g}$ | p) $9.0 \times 10^5 \mu\text{L} \rightarrow \text{L}$ |

That’s all fine and well, but what if you are asked to convert from one prefixed unit to another? It turns out it is essentially the same, but you need to make 2 steps instead of 1. WHEN CONVERTING BETWEEN TWO PREFIXED UNITS, ALWAYS CONVERT THE FIRST PREFIXED UNIT TO THE BASE UNIT AND THEN FROM THE BASE UNIT TO THE SECOND PREFIXED UNIT! Huh? Observe:

How many decigrams are in 739.22 centigrams?

First, write the conversion for centigrams to grams:

Generic unit equation $\rightarrow 1 \text{ c} = 1 \times 10^{-2}$

unit equation $\rightarrow 1 \text{ cg} = 1 \times 10^{-2} \text{ g}$

$$739.22 \text{ cg} \times \frac{1 \times 10^{-2} \text{ g}}{1 \text{ cg}} \quad (\text{notice that the 1 is still in front of the prefixed unit, good check!})$$

now write the conversion for the base unit to the second prefixed unit (dg) and answer:

generic unit equation $\rightarrow 1 \text{ d} = 1 \times 10^{-1}$

unit equation $\rightarrow 1 \text{ dg} = 1 \times 10^{-1} \text{ g}$

$$739.22 \text{ cg} \times \frac{1 \times 10^{-2} \text{ g}}{1 \text{ cg}} \times \frac{1 \text{ dg}}{1 \times 10^{-1} \text{ g}} = 73.922 \text{ dg} \quad (\text{are sig figs correct? Why or why not?})$$

See? It is the same process, but with two steps instead of one. Just remember, ALWAYS convert to the base unit first!

Example: How many milliwatts (mW) are in 1.21 gigawatts (GW)? (again, don't worry about watts, I just wanted to say 1.21 gigawatts, and they are just units)

Generic unit equation $\rightarrow 1 \text{ G} = 1 \times 10^9$
 Generic unit equation $\rightarrow 1 \text{ m} = 1 \times 10^{-3}$

unit equation $\rightarrow 1 \text{ GW} = 1 \times 10^9 \text{ W}$
 unit equation $\rightarrow 1 \text{ mW} = 1 \times 10^{-3} \text{ W}$

$$1.21 \text{ GW} \times \frac{1 \times 10^9 \text{ W}}{1 \text{ GW}} \times \frac{1 \text{ mW}}{1 \times 10^{-3} \text{ W}} = 1.21 \times 10^{12} \text{ mW}$$

Example: The Bohr radius is 53 nanometers. What is that distance in cm?

$$53 \text{ nm} \times \frac{1 \times 10^{-9} \text{ m}}{1 \text{ nm}} \times \frac{1 \text{ cm}}{1 \times 10^{-2} \text{ m}} = 5.3 \times 10^{-6} \text{ cm}$$

Okay, here are some for you to try (again, write out the generic “unit equations”, unit equations, and both conversion factors for at least 6 of these):

- 1) 58216 microliters to dekaliters
- 2) 46.875 terabytes to kilobytes
- 3) How many femtoseconds in 22.16 milliseconds
- 4) If something has a mass of 3.78×10^{-2} megagrams, what is its mass in centigrams?
- 5) 650.89 Gm $\rightarrow$ pm
- 6) 249 cm $\rightarrow$ km
- 7) 45.14 dm $\rightarrow$ Mm
- 8) 570 kg $\rightarrow$ μ g
- 9) 2383.7 Mg $\rightarrow$ mg
- 10) 39.46 μ g $\rightarrow$ cg
- 11) 139.42 pL $\rightarrow$ nL
- 12) 5.23×10^{-4} TL $\rightarrow$ kL

Before I get much further in this “review”, I want to remind you about conversions factors. A conversion fact is simply a way of changing from one unit to another. 1 in = 2.54 cm is a conversion factor because it allows you to convert from inches to centimeters or centimeters to inches. Densities can be thought of as conversion factors: 11.35 g per cm^3 is a conversion factor because it lets you convert from a volume (cm^3) to a mass (g) or vise versa. Anything with two or more units is just a conversion factor. You don't have to know what those units are or what they mean as long as the convert from one thing to another. Finally, I want to remind you that these conversion factors can be written in one of several ways:

$$2.54 \text{ cm} = 1 \text{ in} \qquad 2.54 \text{ cm per in} \qquad \frac{2.54 \text{ cm}}{1 \text{ in}} \qquad 2.54 \frac{\text{cm}}{\text{in}}$$

Each one of these is equivalent to the others and they are all the exact same conversion factor. While all of these are equivalent, I *strongly* suggest that you avoid using the “slash” (the last one). Even though most people intuitively recognize that *cm* on the top of that slash and that the *in* are on the bottom, it is still unbelievably easy to forget that when performing dimensional analysis.

Conversion between systems of measure:

The main difference between this and the metric conversions is that you will not have to memorize these conversions; they will (for the most part) be given to you. (That should make it easier... maybe). So, how do you do this? Let's start off fairly easy and work our way up.

Example: How many liters in 43.125 gallons?

First thing you want to do on ANY conversion problem is write what you are given and what you are asked for. On a problem like this, it may seem to be a waste of time, but as the problems get more complicated, it will be vital in order to keep track of what you are doing.

What is given? 43.125 gallons

What is asked for? Liters

Great, I know where to start, but where do I go from there? I need a conversion factor that will allow me to get from gallons to liters, so I look on my conversion sheet and see that 1 gal = 3.785 L. Looks good to me... I'll give it a try. Remember, that units only cancel if they are on opposite sides of the division bar.

$$43.125 \text{ gal} \times \frac{3.785 \text{ L}}{1 \text{ gal}} = 163.228125 \text{ L}$$

Cool, my answer ended up in L, which is exactly what I needed. Sig fig wise, L and gal both measure volume, but liters are metric and gallons are standard. This means that the conversion factor is NOT infinite and must be taken into account for sig figs. The starting value has 5 s.f. and the conversion has 4 s.f. so the answer must have 4 s.f. The final answer to this is **163.2 L**

If only all of the problems were that straight forward. So how do we deal with problems that are more complicated? In the last problem, we were able to find a direct conversion between gallons and liters. There will not always be a direct conversion, which is when we are really going to have to think.

Example: How many inches are in 0.018 m?

What is given? 0.018 m

What is asked for? inches

Since the 0.018 m is the only value given, we know that we must start there. So we go to our handy dandy conversion table and look for a conversion from meters to inches... and come up empty handed. What now? Now we need to use a little bit of intuition. We look at the conversion table and see that about the only conversion involving inches also involves cm. Can we work with that? Maybe, but in order to use that conversion, we must be in units of either inches or centimeters. We are starting in meters... what can be done? Hmmm, meter and centimeters... That is a metric conversion!! We can do that!

$$0.018 \text{ m} \times \frac{1 \text{ cm}}{1 \times 10^{-2} \text{ m}}$$

Now we are in centimeters and can use the cm to inch conversion!

$$0.018 \text{ m} \times \frac{1 \text{ cm}}{1 \times 10^{-2} \text{ m}} \times \frac{0.3937 \text{ in}}{1 \text{ cm}} = 0.70866 \text{ in}$$

I am in inches and that is what the question asked for! The starting value has 2 s.f., the first conversion has infinite s.f. (both are metric distances), and the second conversion has 4 s.f. (one is standard, one is metric) so the final answer of **0.71 in** has 2 s.f.

Example: What is the volume in fluid ounce of a vat containing 0.023922 megaliters?

What is given? 0.023922 ML

What is asked for? fl. oz.

Again the only place to start is with 0.023922 ML, but again my conversion table has failed me; there is not a ML to fl. oz. conversion anywhere to be found. In fact, ML doesn't appear anywhere in my table, but I see fluid ounces in 2 places: 1 qt = 32 fl.oz. and 1 fl.oz. = 29.57 mL. Hmmm, ML and mL, I think I can do something about that! All I need to do is convert ML to mL, which is a prefix unit to prefix unit metric conversion! (remember, do this in 2 steps, 1st prefix to base, then base to 2nd prefix)

unit equation $\rightarrow 1 \text{ ML} = 1 \times 10^6 \text{ L}$

unit equation $\rightarrow 1 \text{ mL} = 1 \times 10^{-3} \text{ L}$

$$0.023922 \text{ ML} \times \frac{1 \times 10^6 \text{ L}}{1 \text{ ML}} \times \frac{1 \text{ mL}}{1 \times 10^{-3} \text{ L}}$$

Now I am in mL and can use the 1 fl.oz. = 29.57 mL conversion

$$0.023922 \text{ ML} \times \frac{1 \times 10^6 \text{ L}}{1 \text{ ML}} \times \frac{1 \text{ mL}}{1 \times 10^{-3} \text{ L}} \times \frac{1 \text{ fl.oz.}}{29.57 \text{ mL}} = 808995.6037 \text{ fl.oz.}$$

For sig figs, the starting value (which almost always counts for s.f.) has 5 s.f., the 1st and second conversions (each has two metric volumes) have infinite s.f., and the 3rd (metric and standard volumes) has 4 s.f. 4 is less than both 5 and infinity, so the answer, **8.080x10⁵ fl. oz.**, must have 4 s.f. (it must also be in scientific notation, why?)

These conversions are all about doing whatever it takes to get from one unit to another. Above, there was no meter to inch conversion, so we had to see what was available and sort of... build a bridge. This is what you need to practice. Some advice; the process is the same for every one of these so don't get wrapped up in each individual problem, but pay attention to overall thought process.

Practice: (use the "SI Units and Conversion Factors" table in your book, or the conversion table I gave you, to do these)

- 1) Convert 103 kg to ounces
- 2) What is the equivalent of 653 nm in angstroms?
- 3) How many liters are there in 6.375 pints
- 4) 0.75 tons is equal to how many kg?
- 5) How many atomic mass units (amu) are in 3.89266×10^{-17} pounds?
- 6) What is the distance in μm of 8.00 ft?
- 7) How many millimeters in a marathon (26.22 miles)
- 8) What is the volume in nL of 2.336 gallons?
- 9) Given that 1 mL = 0.789 g (for ethanol), convert $7.39 \times 10^7 \text{ m}^3$ of ethanol to tons of ethanol